PNEUMATIC CONVEYING OF MATERIALS OF UNIT

DENSITY IN A THREE-INCH PIPE

A THESIS

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NOMENCLATURE

A - Projected area of a particle in the direction of motion.

C - Concentration of solids in a fluid.

d - Average effective particle diameter.

D - Diameter of conveying line.

f - Fanning friction factor.

g - Local gravitational acceleration.

g_c - Dimensional constant in Newton's second law.

G - Mass velocity.

h - Head loss.

L - Length of pipe line.

R - Specific loading, mass rate of flow of solids per mass rate of flow of air.

u - Velocity.

V - Volume of a solid particle.

W - Mass rate of flow.

α - Specific pressure drop, mixture pressure drop/air pressure drop.

ρ - Density.

μ - Viscosity.

Subscripts:

a - Air.

ds - Dispersed solids.

m - Mixture of air and solids.

s - Solids.

SUMMARY

Techniques utilizing the operation of pneumatic conveying have been in use for almost half a century, but the literature on the subject is very limited. As a result of such a situation the design of conveyors has been carried out on a highly empirical basis subject to much inefficiency. Various correlations based on the flow of solids with air have been proposed in the literature, but there is little common agreement among the writers. It is apparent upon consideration of these works that certain underlying factors, notably those pertaining to the establishment of the equilibrium solid velocity, have not been given their full significance. Not all writers have ignored the effects of such factors, but in many cases the treatment of them lends uncertainty to the results presented. In the light of these facts the present investigation has been undertaken to reexamine the general problem of pneumatic conveying in order to establish the importance of the various factors and also to attempt the formulation of a simplified relationship expressing the frictional losses in pneumatic conveying. Such a relationship would be valuable to the engineer who desires to design such a system, especially if the equation were expressed in a familiar form such as the Fanning relationship. Due to the limitations imposed by time the scope of the investigation was limited to the following considerations:

(a) The effective length of pipe to achieve acceleration.

(b) The effect of the roughness of the pipe on the pressure drop.

(c) The effect of size and shape of particles of like density

on the pressure drop.

(d) The effect of the rate of feeding of solids and the rate of air flow on the pressure drop.

(e) The determination of the relative velocity of the air to that of the solids in conveying.

(f) The minimum air velocities to convey material fed at a given rate.

The materials chosen for use in the study are considered to be representative of the types usually encountered in commercial conveying installations with the exception that, due to considerations of the physical limitations of the conveying system, the particles specific gravities were limited to a range between 1.0 and 1.2. However, the range studied includes a high percentage of the densities encountered in commercial practice, especially where grains are the conveyed material. The principal part of the data for this study was obtained on soybeans, cottonseed and Tenite, cellulose butyrate acetate molding pellets. These materials exhibited a number of different shapes and covered a size range of from 0.04 inches to 0.25 inches diameter.

The results of this study demonstrate the extreme necessity for insuring the attainment of velocity equilibrium in studies on the flow of solids in air streams. As is demonstrated by measurements over the section of pipe through which particles were being accelerated the acceleration losses are appreciable in comparison to the frictional losses. Hence, any study of the frictional effects of air-solid flow in pipes must be carried on in test sections where the acceleration of particles is virtually zero, except for the acceleration of individual particles which have been slowed by their contact with the pipe wall.

Measurements of the pressure drops encountered in the test section for different rates of solid and air flow have been obtained. These measurements have been carried out from the maximum rate possible with the equipment used to the minimum rate defined by the tendency of the solid material to settle out of the air stream at a particular velocity. The data have been correlated in a modified friction factor - Reynolds' number relationships through the use of a correlating factor based on the ratio of the density of the air-solid mixture to the density of the solid material being conveyed. This has resulted in an equation which fully expresses the flow of air-solid mixtures for the conveyance of materials in the density range covered. Supplementary data on material of higher specific gravity than that of the material used in the main part of the investigation indicates an exponential dependence of the correlating factor upon the density of the solid. However, the type of flow encountered in the conveying of this material of high density did not permit the obtaining of enough accurate data to completely substantiate this proposal. No effects of particles size and shape could be determined in the correlation, which indicates that, at least for values of the ratio of particle size to pipe diameter usually encountered in conveying, these variables are of little importance.

The minimum flow of air necessary to convey solid material at a given feed rate has been determined for the materials of the study. The data have been expressed as a relationship between the modified friction factor and the modified Reynolds' number and thus the conditions at which the solids would tend to settle out of the air stream have been defined.

Resistance studies were made on the flow of cottonseed in an artificially roughened pipe. The data demonstrate the extreme dependence of the resultant pressure drop on the degree of roughness of the pipe, for, with only a slight degree of roughness introduced by the conveying of abrasive materials, the resultant pressure drop with a given flow of solids and air is greatly increased by the roughness of the pipe. In addition to the increase in pressure drop noted the slope of the curves of pressure drop versus air flow rates with parameter feed rate was decreased greatly.

The true velocities of the solids have been measured by means of a technique utilizing the high speed motion picture camera. Data are presented at a number of different air and solid rates in the conveying of Tenite. The velocities thus determined demonstrate that the relative velocity between the air and the solid is constant for a given solid feed rate and that an increase in the loading of the air stream causes an increase in the relative velocity.

PNEUMATIC CONVEYING OF MATERIALS OF UNIT

DENSITY IN A THREE-INCH PIPE

INTRODUCTION

Industrial Importance

Pneumatic conveying, the process of moving solid materials through pipes by means of air, has been in use for years, particularly in Europe where it has been found to be an excellent means to unload grain from ships. Its use in this country in the past has not been too widespread, partially because of our economic history. The United States has never been a large importer of grain and as a result the need for rapid unloading of granular materials did not lend impetus to the use of the technique here. However, there has been a gradual growth here in this use, particularly in the grain industry. Since the development of the technique of fluidized catalytic cracking during World War II, much greater interest has been shown in it by chemical engineers.

Basically, pneumatic conveying is not the cheapest method of transporting material from one point to another. The cost of power alone is considerably higher than in usual methods of moving materials such as belt or bucket conveyors and like procedures. However, when the materials to be moved are expensive, or are likely to be hazardous to workers, pneumatic conveying is frequently the best solution. In addition, a properly designed pneumatic conveying system requires little attention and therefore the high proportion of operating expenses attributable to labor has placed pneumatic conveying in a more favorable position.

An interesting case where pneumatic conveying has been used is in the construction of Boulder Dam. There a Fuller Company "Fluxo" pressure system was used to transport dry cement about one mile in a single stage at rates up to seventy-six tons per hour. This example gives an idea of the possibilities of the procedure with regard to size of the installation, although in fact some installations run at much higher rates.

In the future, as fluidized techniques grow and as the use of pneumatic transportation to move materials from one point to another becomes more widespread the chemical engineer will find himself more intimately concerned with the design of equipment to carry out the desired process. As a result a program of study to relate the principles of conveying to the existing techniques in fluid flow is felt to be needed.

Most of the work done in the field of pneumatic transportation up to the last five years has been done in England and Germany, although the quantity of relevant published data is quite small. There have been a large number of papers published describing particular installations in a qualitative manner which, while informative, have not served to establish any basic relationships suitable for design calculations on proposed systems.

Review of Literature

Of the papers published prior to 1940 only those of Cramp (2), Gasterstadt (6) and Wood and Bailey (12) have been comprehensive in their treatment.

Cramp (2) investigated the friction effects on wheat in a vertical pipe and discussed thoroughly the fundamentals of the flow. Of particular importance is his summary of the force terms which must be considered when dealing with the momentum transfer between the air and the solid. According to Cramp these terms are:

1. The differential pressure on the two ends of the column of air multiplied by the area of the pipe.

2. The friction between the pipe and the material being conveyed.

3. In vertical pipes, the force required to support the column of material.

4. The friction of the air on the pipe.

5. The force required to support and accelerate the air.

6. The force required to accelerate the material.

It is further demonstrated how these various force terms may be calculated and applied to a problem in conveyor design.

Cramp (2) also describes an experimental procedure whereby the horizontal conveying velocity may be determined. An equation involving an arbitrary constant for a particular material is proposed. In the description of the procedure for determination of this constant he recommends a three foot long duct on the positive side of a blower, material to be introduced at the blower end of the duct. From measurements of the distance the material is blown from the open end, the velocity may be calculated. (It is the author's finding that the three foot duct recommended by Cramp is not long enough for the material to reach an equilibrium velocity with relation to the velocity of the air stream.)

Gasterstadt (6), in tests on wheat, concluded that a simple relationship might be possible to relate the pressure drop for a single phase to the pressure drop when more than one phase is flowing. He defined a dimensionless factor α as the ratio of mixture pressure drop to air pressure drop for air at the same velocity and proposed that a linear relationship existed between α and the specific loading. (Specific loading may be defined as the ratio of the solid rate of flow to gas rate of flow and is denoted by the letter R.)

Segler is reported by Vogt and White (11) to have verified the findings of Gasterstadt in the conveying of wheat by air.

Wood and Bailey (12) present a very detailed study of a system utilizing an injector as the motive agent. Their primary purpose was the determination of the optimum position for the injector; however, they cover the basic principles in the flow of solid-air mixtures very well. Unfortunately their pressure drop measurements included the pressure differential across the injector, which makes it difficult to compare the results of their study with the present one in more than a qualitative manner. Among the conclusions they present are:

1. The optimum position for an injector in a conveying line is in the middle of the line.

2. The use of a conical diffusor at the outlet of a conveyor system improves the performance of the conveyor by the conversion of kinetic energy to pressure energy by the reduction of the velocity. (The writer doubts that the diffusor used by Wood and Bailey decreased the velocity of the moving solid to any great degree and believes that the major improvement occurred by the decrease in velocity of the air.)

3. The path of a particle in the pipe appears to be a series of

leaps for, as the particle is picked up by the air and given motion down the pipe, gravitation causes it to fall back to the bottom of the pipe. This is usually referred to as "saltation".

It was also noted during the present study that the particles tend to strike the top of the pipe, particularly when their concentration is low. The motion seems to be of random nature, especially when the air velocity is increased at a given solid feed rate.

Davis (3) proposed a formula for the minimum velocity required to pick up a particle from the bottom of a pipe which is

$$u = \sqrt{\frac{V(\rho_s - \rho_g)\, 2g}{\rho_g\, A}} \qquad (1)$$

where V/A is the ratio of the volume of the particle to its cross-sectional area, g is the gravitational acceleration constant and ρ_g and ρ_s are the densities of the gas and particle, respectively. However, since this minimum velocity is that in the immediate vicinity of the particle, it is quite difficult to measure. Furthermore, Davis's equation fails to take into account the influence of neighboring particles, which undoubtedly have considerable effect.

Vogt and White (11), following the ideas of Gesterstadt (6), propose the use of a dimensionless term which is the relative pressure previously defined. They found their data to be correlated by the following relationship:

$$\alpha - 1 = A \left(\frac{D}{d}\right)^2 \left(\frac{\rho_g}{\rho_s} \cdot \frac{R}{Re}\right)^k \qquad (2)$$

where D and d are the diameters of the pipe and particles, respectively, $R = w_s/w_g$, the ratio of the weight of particles to the weight of gas used to convey them, Re is Reynold's number and, except for A and k, the other quantities are as previously defined. The values of A and k are expressed as empirical functions of the dimensionless group

$$\sqrt{(1/3)(\rho_s - \rho_g)\rho_g d^3}/\mu$$

which is the product of the Reynolds number and the drag coefficient of a spherical particle under free settling conditions. However, Vogt and White did not find a linear relation between α and specific loading as was the case with Gasterstadt's data. The report of these investigators discusses at length the theory of pneumatic conveying and the factors influencing the flow of two phase systems of solids and gases. Their experimental work was limited to the flow of materials in a loop constructed of one-half inch pipe. The horizontal section was situated at a point nine feet from a large radius bend and there is some question as to whether the flow had reached equilibrium before the test section was reached. Vogt and White state in their discussion that a distance of nine feet from the feed point was not a sufficient length to reach equilibrium under all conditions studied. No data are presented confirming this fact and it is not known to what extent the data they report are affected by non-equilibrium conditions.

Belden and Kassel (1) state that Vogt and White placed an apparently incorrect dependence on the ratio of pipe diameter to particle diameter. In their study on the movement of cracking catalyst in a vertical pipe, they propose a separation of the friction and static terms of the pressure drop. A correlation of the friction pressure drop in a generalized Fanning formula is proposed and used in the equation

$$\frac{dp}{dL} = \frac{2f(G_g + G_s)\,u}{g_c D} \quad , \qquad (3)$$

where the G's represent the mass velocities of the gas and solids as identified by subscripts. Unfortunately, Belden and Kassel's results are open to question because of their correction of acceleration losses on what they call a "somewhat speculative basis." Also their statement that the correction is small has been found in the present study to be incorrect.

Farber (5) has investigated the flow characteristics of a mixture of fine particles in a combined horizontal and vertical system. Several types of nozzles for feeding solids were investigated and quantitative information was presented on the type of nozzle yielding the most uniform type of mixing. Unfortunately, Farber does not include a diagram of the nozzle and it is difficult to discern in the reproduction. He also presents qualitative observations on the flow pattern for mixtures of various sized particles.

Hariu and Molstad (7) present a treatment covering the transport of solids through vertical lines. Direct measurements of the density of the dispersed solids were made, permitting the calculation of the average particle velocity. From their measurements the average particle velocity is approximately 50 % of the gas velocity; however, a figure somewhat higher may be obtained if the material reaches full equilibrium with the air.

Lapple (9) proposed the use of a modified form of the Fanning equation such that

$$\Delta P = \left(\frac{4f \; L \; G_a^2}{2 \; g_c D} \right) (1 + R) \left(\frac{1}{\rho_a} + \frac{R}{\rho_s} \right) \quad , \tag{4}$$

where f is the Fanning friction factor, L is the length of pipe, D is the pipe diameter, ρ_a and ρ_s are the densities of the air and solid, respectively, and R is the specific loading, the ratio of the weight of solid to the weight of air in a given time. He states that the pressure drops predicted on this basis will be somewhat lower than the actual drop encountered in the system. The friction factors are determined from the charts of f vs. Re, using the Reynolds number calculated according to the following:

$$Re = \frac{D \; Ga}{\mu_a} \; (1 + R) \quad . \tag{5}$$

Objectives of Present Investigation

It is evident from the foregoing that the amount of work that has been done with regard to pneumatic conveying is limited. Furthermore, it is clear that there is no general agreement as to the form of equation which is sufficiently general to show precisely how the variables involved in the problem should be correlated. It is the object of this thesis to reexamine the general problem of pneumatic flow and derive a relationship which better expresses the law of pneumatic conveying. At the same time, the projected scope of study includes considerations which would lead to simplifications and to obtain formulas of practical utility.

It was early recognized that the great number of variables involved could not be separated or studied groupwise in the time allotted for the investigation. Consequently, the scope of the research here described was limited to the following considerations:

(a) The effective length of pipe to achieve acceleration.

(b) The effect of the roughness of the pipe on the pressure drop.

(c) The effect of size and shape of particles of like density on the pressure drop.

(d) The effect of the rate of feeding of solids and the rate of air flow on the pressure drop.

(e) The determination of the relative velocity of the air to that of the solids in conveying.

(f) The minimum air velocities to convey material fed at a given rate.

EQUIPMENT AND MATERIALS

Description of Equipment

The essential features of the conveying system used in this study are indicated on Figure 1.

The feeder was a screw type consisting of a feed hopper, helical screw and driving mechanism. The rate of feed was controlled by adjustment of a Master Speed Ranger used as the main drive. The solid material fed by the feeder dropped through a vertical pipe into the inlet nozzle of the conveyor line.

The inlet nozzle of the conveyor line consisted of a truncated cone ending in a cylindrical section, which fit into the end of the conveyor line. The solid material entered the nozzle through a hole cut in the cylindrical section next to the junction of the conical section. The nozzle served the purpose of decreasing the entrance loss and of facilitating the initial acceleration of the solid material.

The conveyor line was a length of three-inch standard pipe, selected for straightness, terminating in a glass section. The glass section was flange connected to the steel pipe to facilitate alignment of the sections.

Separation of the solids from the air was accomplished in a centrifugal separator. Due to the size range of the particles studied it was unnecessary to use a cyclone separator with the resultant high pressure loss. The efficiency of the separator on the materials studied was very good. However, the hard materials used to roughen the conveying

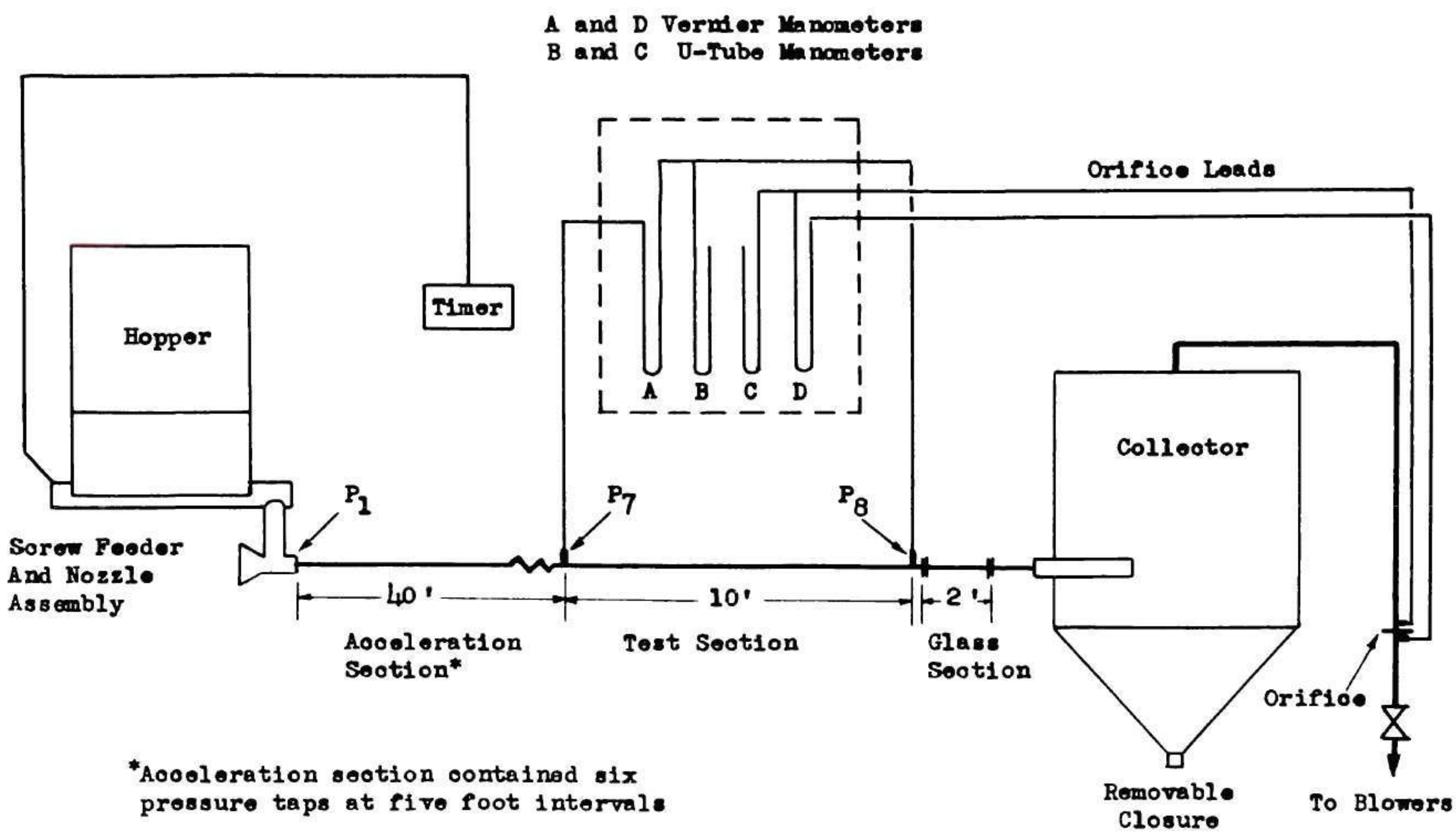

Figure 1. Schematic Diagram of Experimental Apparatus.

line in a later part of the investigation exhibited a tendency to break down in the line into fine particles which the separator could not remove from the air.

The blowers used were Buffalo Forge Company Type 2-RE with eighteen inch impellers. They were connected in series to furnish a sufficient vacuum to operate the system (see discussion in the section, "Experimental Procedure"). Flow of air was regulated by means of a gate valve situated on the low pressure side of the blower system.

Instrumentation

The flow rate of the air was measured by use of a standard orifice calibrated in accordance with the specifications of Stearns, Jackson, Johnson and Larson (10). Orifice coefficients as developed by these investigators were used. The air was metered in the line between the blower and the separator as the orifice coefficients would not apply in the presence of a solid phase with the air. Calming sections in accordance with the specifications of Stearns, *et al.*, were provided in the orifice installation.

Pressure taps were installed as is noted in Figure 1, the last tap being situated one foot ahead of the glass section. In conjunction with this tap a pitot tube was installed.

Differential pressures across the orifice, pitot-static and test section were measured by means of micro-manometers constructed from machinist height gauges. These gauges enabled readings of 0.001 inches of manometer fluid. However, during conveying runs such accuracy was not possible due to slight pulsations encountered, and the last decimal place was not used in making calculations. Static pressures were measured by simple U-tube manometers. In all manometers the fluid used was distilled water, in which a small amount of dye was dissolved.

Materials Used in the Investigation

The materials used in this study were chosen primarily for their size and shape characteristics. The physical data on the materials is listed in Table I.

The Tenite material was in the form of a roughly cubical shape intermixed with some irregular particles. During some preliminary runs the material was degraded to a certain degree, which tended to round off the corners and split some of the cubes into approximate halves and quarters. This degradation resulted in a sufficient amount of fine material to justify the screening of the entire lot. As may be noted in Table I, the material used in the final runs is 87.5% by weight of size greater than 0.0661 and the maximum dimension of any particle is 0.120 inches. This last was determined by micrometer measurement of a number of the particles retained on the largest screen.

The fine material screened from the bulk of the Tenite was analyzed as noted in Table I and later used to determine a few check points.

The soybeans used were in the condition as received from the supplier with the exception that a small amount of foreign material such as stems and leaves were removed by screening. The characteristics of the material were as given in Table I. The size determined by the direct measurement of a large number of beans with a micrometer. These beans had a typical bean shape similar to ordinary navy beans with the same sort

TABLE I.

Characteristics of Materials Studied

TENITE (COARSE)

Specific gravity	1.13
Particle density	70.5 lb./cu. ft.
Bulk density	45.3 lb./cu. ft.
Percent voids	35.7

Approximate angle of slide	28°
Approximate angle of stop	20°

Sieve Analysis

Retained on No. 8	(0.0939 in.)	54.3 %
Retained on No. 12	(0.0661 in.)	33.2 %
Retained on No. 16	(0.0469 in.)	8.5 %
Retained on No. 18	(0.0394 in.)	2.0 %
Less than	(0.0394 in.)	1.9 %

Maximum size of any particle 0.120 in.

SOYBEANS

Specific gravity	1.17
Particle density	73.0 lb./cu. ft.
Bulk density	47.2 lb./cu. ft.
Percent voids	34.4

Approximate angle of slide	25°
Approximate angle of stop	18°

Size:	Length	0.299 in.
	Width	0.261 in.
	Thickness	.220 in.

TABLE I. (Continued)

COTTONSEED

Specific gravity	1.0	(Determined by flotation)
Particle density	62.3	lb./cu. ft.
Bulk density	32.2	lb./cu. ft.
Percent voids	48.3	
Approximate angle of slide	30°	
Approximate angle of stop	19°	

Size: Length 0.336 in.
Thickness 0.176 in.

TENITE (FINE)

Specific gravity	1.13	
Particle density	70.5	lb./cu. ft.
Bulk density	41.1	lb./cu. ft.
Percent voids	41.8	
Approximate angle of slide	31°	
Approximate angle of stop	21°	

Sieve analysis

Retained on No. 10	(0.0787 in.)	2.2 %
Retained on No. 12	(0.0661 in.)	7.4 %
Retained on No. 16	(0.0469 in.)	28.7 %
Retained on No. 20	(0.0331 in.)	26.4 %
Retained on No. 40	(0.0165 in.)	25.3 %
Retained on No. 50	(0.0117 in.)	5.9 %
Retained on No. 60	(0.0098 in.)	1.8 %
Retained on No. 80	(0.0070 in.)	1.4 %
Retained on No. 120	(0.0049 in.)	0.5 %
Less than	(0.0049 in.)	0.5 %

ALUNDUM

Specific gravity	1.82
Particle density	113.3 lb./cu. ft.
Porosity	42 - 45 %
Size (Diameter)	0.3293 in.

of smooth surface. The breakage of the beans in transportation was not measured quantitatively, but observations indicated that the total number of beans broken during all runs was no more than 10 to 15 % of the total. No effect could be noted as a result of the breakage.

The cottonseed, as received from the supplier, had mixed with it a fairly large amount of loose cotton linters. The seeds were a process grade used as a source of cottonseed oil and had been run through the delinting machinery at the mill. The loose linters were a result of the inefficiency of the process for separating removed linters from the seeds. Considerable effort was expended to remove these loose linters from the bulk of the seed, for it was noted in trial runs that the linters tended to clump together in the separator, causing large fluctuation in the flow on subsequent runs. Various methods for the removal of these loose linters were attempted, with the final conclusion that screening with a mesh just too fine to pass the seeds was most satisfactory. Although the process was rather time consuming it was found that the removal of the majority of this lint contributed very much to steadying the flow of the seeds in the conveyor.

In addition to the loose cotton linters mentioned above, the seed had attached to the ends a large amount of linter of fiber length between 1/16 and 1/8 inch which could not be removed by any practical method in keeping with the subsequent use of the seed. The removal is possible by use of a strong caustic solution; however, this leaves the seed in an undesirable state for conveying studies.

The measured properties of the cottonseed are given in Table II.

Like soybeans, the size was determined by micrometer measurement of a large number of seeds. However, only two axes were measured on each particle, due to the shape. The seeds are very close to the classical tear-drop shape when they are cleaned of all linters, but with linters attached they roughly resemble cylinders with a length/diameter ratio of about two. The skin of the seed is moderately smooth, though slightly irregularly wrinkled.

The reported angles of slide for the three materials were determined experimentally. A flat metal plate twenty inches long was rested on a flat surface. A small quantity of material was placed near one end of the plate and that end was raised until the material just began to slide. The end was then lowered until the sliding ceased. Both angles were measured and the average of five determinations is reported.

The alundum spheres for which some data are reported were relatively uniform in size. Their surface was porous and rather gritty, which undoubtedly contributed to their behaviour in conveying. The physical properties are as given in Table I. The diameters of a large number of particles were obtained by direct measurement and the average weight per particle determined by weighing in groups of five. From these measurements the particle density and specific gravity were determined.

The effective particle density of the cottonseed was impossible to obtain by the pycnometer technique, which was used for the soybeans and Tenite. The linters attached to the ends of the seed caused the effective density to be lower than the actual particle density. Therefore the flotation technique was tried and it was discovered that the seeds could be immersed long enough for a determination before the mass of linters became saturated with the test liquid. It was found that the majority of seeds just tended to float in distilled water until the linter mass was saturated and the seeds sank. It was deduced from this that the effective density was very close to that of water and thus the specific gravity was reported as 1.0.

EXPERIMENTAL PROCEDURE

Prior to the operation of the equipment to obtain data for the study, a number of runs were made to determine the characteristics of the feeder mechanism. It was initially felt that a calibration curve of feed rate versus speed regulator setting would be the best procedure to determine the correct value of the feed rate for use in evaluating the data. In addition, since the head of material in the feed hopper was continuously decreasing as a run progressed, it was felt that the feed rate might vary with time. However, measurements of the feed rate proved that as long as the level of the material did not drop below the upper rectangular section of the feeder the feed rate remained uniform. This gave a constant feed time of about five minutes at the maximum speed of the screw at that time. A typical curve illustrating this point is presented in Appendix I.

During the calibration of the feeder the conveyor system was operated to allow the pipe to attain an equilibrium state of roughness for the materials to be used. The pipe was substantially unaffected by this procedure, indicating that the materials used had no appreciable effect on the roughness. It was noted, however, that there was a slight deposit of Tenite on the walls but that did not seem to interfere with the flow.

Due to an erroneous assumption (based on the literature) of the distance required to accelerate the material, the feed point was placed approximately twenty pipe diameters from the first pressure tap. The test section was then calibrated with air and a series of runs were made on Tenite at various loadings. During this stage of the operations the air was metered by measuring the pressure differential across a section of the line leading from the separator to the blower. This meter section was calibrated against the orifice previously described, which, with a single blower in the system, produced too great a permanent loss for use when conveying solid material.

After the first series of runs were completed it was noted that the original assumption of equilibrium might be in error and measurements of the pressure gradient from a number of points to the final test section pressure tap supported this view. The pipe was then lengthened to allow fifty diameters ahead of the test section and further measurements of the pressure gradient were made. These indicated that equilibrium had still not been attained and a long section of pipe was installed. The final length allowed for attainment of equilibrium was one hundred and thirty diameters, and measurements at five foot intervals indicated that equilibrium was attained in a distance of about one hundred diameters. Further results of the equilibrium study are presented in the discussion.

At the same time that the long section of pipe was installed another blower was connected in series with the original to supply the increased suction required by the longer conveying line. The increased vacuum obtainable with the second blower made it possible to operate, using the orifice to meter the air. The maximum feed rate of the feeder was increased by varying the pulley ratio.

The system was again calibrated with air alone and another series of runs were made. After this series the calibration was checked and found to be different from the original check. Investigation disclosed that the increase in vacuum of the system had caused a leak to develop in the separator. The entire system was then rechecked for leaks and all possible points of leakage sealed with appropriate sealing agents.

At this time the feeder was rechecked and the fact that the feed rate was constant for a given air rate through the system for as long as two minutes at high feed rates was substantiated, even though the material level had dropped below the rectangular section of the hopper. This fact made possible higher rates of operation than formerly with less material. It had also been observed in previous runs that the flow became steady in less than ten seconds and it therefore became possible to make runs almost as rapidly as all the instruments could be adjusted and read.

The final series of runs were then made on Tenite, soybeans and cottonseed. During this set of runs the use of the pitot tube was discontinued due to its tendency to become clogged and knocked about by the impact of the particles. (Wood and Bailey (12) report this difficulty in their work; however, Gasterstadt in a critique of their paper reports that he had no difficulty with the use of a pitot tube in his work.) Periodically, the pressure drop for air alone in the test section was checked against the original calibration and was found to be unchanged. Data for this series of runs have been tabulated in Table V of Appendix II.

In order to determine the effect of material of higher density and to roughen the pipe for the studies on pipe roughness, a quantity of cinders and crushed granite was procured. After separating the material into the sizes which could best be handled by the feeder and separator, a series of runs were made using these materials. A check of the air calibration curve showed consistently higher pressure drops than had been previously encountered. The measurements obtained during these runs were inconsistent due to the changing condition of the pipe and therefore are not included in the tabulated data.

A set of runs were then made, using cottonseed as the material, to show the effect of roughness on the pressure drops encountered in conveying.

Finally, to determine the characteristics of a material of higher density a set of runs were made using alundum spheres. Due to the highly abrasive qualities of this material the glass section was not used, but flow observations were made from the inlet point.

Photographic studies were made utilizing a high speed motion picture camera. A motion picture was made on each of four loadings of Tenite and air. These pictures were examined in a viewer and the particle velocities were computed from the film speed calibration.

DISCUSSION OF RESULTS

The results of this investigation include data on the behavior of the flow of solids having a specific gravity of approximately 1.0. As shown in the preceding chapter, the material included a number of different sizes and shapes. By a procedure discussed below, it has been possible to obtain a simple relation, which adequately describes the law of pneumatic conveying. While the work was done with a three-inch diameter pipe, the results are nevertheless of some generality and should apply to any diameter of pipe having a similar degree of roughness.

When solid materials are introduced into a moving stream of air, the drag of the air on the particles imposes unbalanced forces on the particles, causing them to be accelerated. Due to the high degree of turbulence set up by the particles the resultant particle motion is randomly distributed in the general direction of the mass flow of air. At this point of entry of the solids the slip (the relative velocity of the air with respect to the particles) is at its maximum value. As the particles are accelerated, the slip gradually approaches an equilibrium value until the average force acting on a particle just balances the frictional force resisting motion. At this point, the flow assumes a steady state and the pressure drop in the line is a measure of the energy losses due to friction between the pipe and the air-solid mixture. Of course, as the air continues to expand there is a corresponding increase in air velocity and a resulting tendency to accelerate the particles. However, to partially offset this condition, the density of a gas decreases as it undergoes expansion and the resultant drag would tend to decrease.

Wood and Bailey (12) found in their tests on linseed that the pressure gradient is steepest at the solids inlet and gradually decreases to a steady value at a distance of thirty to forty pipe diameters from the solids inlet. From this, the inference has been drawn that the distance required for a given material to reach equilibrium is a constant and is independent of loading. The data of this study indicate that the distance required to complete the momentum transfer increases with loading as is shown on Figure 2. However, the equilibrium distance is more a function of solid introduction technique than of loading, for, if the particles are given a high initial velocity, they would tend to reach equilibrium more rapidly.

Necessary Pipe Length for Equilibrium Velocities

It was found early in this investigation that the accepted entrance length - that is, the length of duct preceding the test section - had to be much more than the accepted thirty to forty diameters usually considered as sufficient to assure homogeneous distribution of the suspended particles.[1]

To determine the effect of entrance length on the pressure drop, pressure readings were made at various points following the point at which the material was introduced. The results are shown in Figure 2 for a light and heavy loading of Tenite. It is clear from these curves that the lighter loading (19 lb./min.) begins to show a linear trend in

[1] I am indebted to Prof. H. C. Lewis for the suggestion to investigate this aspect of the general problem.

pressure drop at about sixty pipe diameters from the entrance. The higher loading (53.8 lb./min.) requires about one hundred pipe diameters to achieve the expected linear drop. It is to be noted that the pressure gradient is initially quite steep and, had runs been made within the region of velocity instability, the results would have been seriously affected and would have failed to give an accurate picture of the flow in the horizontal section under investigation. Undoubtedly, the analyses of earlier investigators, notably Vogt and White (11), are in error because of neglect in supplying an adequate entrance section preceding the section under investigation. The widespread discrepancies in reported results can also be in part attributed to failure to appreciate the need for testing the pipe sections of sufficient length to assure uniform motion of all the particles.

Coincidental with the results here reported on the length of entrance section required, it was also established that when the particles achieved a uniform velocity the pressure drop along the test section was linear. It was this fact which permitted us to make the form of analysis of resistance to motion discussed in a subsequent section of this chapter.

Resistance Studies

After the establishment of the fact that an entrance length of more than one hundred pipe diameters is sufficient to accelerate particles of specific gravity 1.0 to a uniform velocity, the investigation of the pressure drop under different conditions in the test section was undertaken. The data obtained with regard to the several materials investigated in relation to rate of feed and air volume are included in Appendix II. A portion of these data are shown plotted in Figures 3, 4, and 5. These figures indicate that over the range of velocities and feeds investigated, the pressure drop and air flow plot is a straight line on log-log grid. For each material the curves for different feed rates are virtually parallel to each other, the intercepts being arithmetically distributed, that is, doubling the feed doubles the pressure drop.

It is necessary to point out that there are situations where the volume of particles per unit volume of air is the same, since the curves shown in Figures 3 to 5 cover a fairly wide range of air volumes. However, this fact in no way leads to identical pressure drops in such instances, since the velocities of flow are not the same.

The nature of the pressure drops shown in Figures 3 to 5 are similar to those obtained in hydraulic dredge work as reported by Dent (4) and more recently by Howard (8). It is curious that investigators of pneumatic conveying problems have not availed themselves of the vast amount of information on hand with regard to the laws of hydraulic conveying. Thus, Howard gives the formula for the head drop, h_f, in a four inch dredge line conveying sand as

$$h_f = K C v^n \quad , \tag{6}$$

where C is the concentration of solids, v is the velocity of the fluid, and K is a constant. Our results indicate a similar form of relationship, as is readily interpreted from an inspection of the curves referred to.

Minimum Conveying Velocities

Before undertaking the analyses of the results of pressure drop, it is well to examine the nature of the conveying phenomenon itself. It is clear that for every feed there is a minimum air velocity at which the conveying takes place. However, before this point is reached, the material begins to behave peculiarly, moving as if it were in a sort of "slugging" motion. The actual velocity at which no material flows cannot be determined since "slugging" interferes. For our purposes it was necessary to establish some procedure for determining just when there was a departure from true conveying, and motion consisted actually of a bed motion due to settled particles and subsequent slugging. The mechanism of slugging is as follows; when a buildup of material occurs, the cross-sectional area of the pipe is decreased, resulting in an increased air velocity with continued conveying. However, if the rate of increase of the pile of material is greater than the rate at which it can be carried away, the difference is deposited along the bottom of the pipe, eventually for the entire length. Finally the constriction at the throat cuts off all air and the flow virtually ceases, practically the entire pipe being full of solid material. If the air velocity is then increased the pipe will gradually be cleaned out (so long as the material does not agglomerate). The material is removed first at the entrance to the pipe, rising into the moving stream from an escarpment which travels down the pipe. This phenomenon was deduced by Davis (3) to be an explanation for the mechanism by which the solid particles are picked up by the air. He proposed that the unbalanced thrust tending to lift the grain into the stream was due to the velocity head of the impinging gas.

In this work it was chosen to define the initiation of the clogging condition as that point at which slugging flow was observed in the glass observation section. The curves of Δp versus air rate show a certain tendency in the points through which they are drawn. They present a suggestion of a break in the middle position indicating that a different type of flow might take over when slugging begins establishing a new pressure drop-air rate curve. It is observed that when slugging becomes visually apparent the flow becomes discontinuous to a marked degree. This phenomenon has not been discussed in the literature; however, it has been observed in connection with work on fluidized catalysts. The present thought on the matter is that as material buildup occurs the resultant increase in air velocity causes a slug of material to be carried off. Farbar (5) reports in clearing a line full of fine catalyst, that the line cleared easily, the whole mass moving as a plug of fluidized material. Whether these phenomena indicate a change in the type of flow remains for further investigation to prove. Unfortunately it was not possible with the equipment available to explore this region. If data were available it is believed that the intersection of the resistance curve in slugging regime would establish a critical point which might be called the settling velocity point.

Based on the observation of slugging in the glass section mentioned above, the settling velocities have been calculated and tabulated in Table II. The values of minimum velocity are considerably higher than those calculated from Davis' (3) formula for the minimum velocity to pick up a particle from the bottom of the pipe. Wood and Bailey (12) report the same discrepancy, as has been previously discussed.

TABLE II.

Calculated Minimum Conveying Velocities

Material	Feed Rate, Lb./Min.	Air Rate, Lb./Min.	Air Rate,* Cu. Ft./Min.	Air Velocity,* Ft./Sec.
Soybeans	12.6	12.9	179	56
	25.5	14.2	197	62
	32.5	14.8	206	65
	62.5	15.8	232	73
Tenite	11.6	12.0	167	53
	17.3	13.2	183	58
	38.0	15.2	211	67
	53.4	16.1	225	71
Cottonseed	12.0	15.8	220	69
	22.4	16.3	226	71
	40.8	16.9	235	74

*based on air density of 0.072 lb./ cu. ft. and pipe area of 0.053 sq. ft.

Correlation of Observed Data

The curves shown in Figures 3 to 5 consist of a family of straight lines, each of which may be represented by an equation of the form

$$\frac{h}{L} = k\,W_a^n \quad , \qquad (7)$$

where h/L represents the pressure drop per unit length of pipe, W the weight of air moved per unit time and k and n are constants. For the particular three inch pipe used, the slopes of the lines are substantially the same except at very low feed rates and hence, n is constant. Later, it will be shown that the roughness of the pipe materially affects the value of n. In other words, the effect of wall roughness is even more appreciable when material is conveyed as when air alone is moved. This effect had not been suspected and is perhaps a further contributing factor to the differences existing among the data of earlier investigators in the field of pneumatic conveying.

It was found that the constant k is a function of the solids per unit time, W_s , and that k could be represented by an equation of the form

$$k = a + bW_s \quad , \qquad (8)$$

where a and b are constants. Thus the pressure drop is seen to be composed of two parts, the first representing the resistance of the pipe without material flowing, and the second part the resistance caused by the flow of the material itself:

$$\frac{h}{L} = a\,W_a^n + b\,W_s\,W_a^n \quad . \qquad (9)$$

It was found for soybeans that the pressure drop, expressed in inches of water gauge per hundred feet of pipe, is given approximately by the equation

$$h_{100} = 0.079\,W_a^{1.6} + 0.0011\,W_s\,W_a^{1.6} \quad , \qquad (10)$$

where W_s is the rate of solid feed and W_a is the rate of air flow both expressed in pounds per minute. Note that the second part of equation

(10) corresponds with the expression developed by Howard (8) for movement of sand in a four inch dredge line. Howard found that the value of n varied greatly with the amount of solids. This variation was not found to be as serious in the case of pneumatic conveying as long as the material was moving faster than the critical slugging velocity discussed earlier. In dredge work, as Howard has shown, the bulk of the solids actually flow along the bottom of the dredge line, so that a value of n dependent upon the amount present is self evident. It should be noted that, over the range studied here, the value of n did vary slightly and an average value has been used in equation (10). Also the effect of the presence of particles on n prevents the equation from being used for the condition when no solid material is flowing. The maximum deviation of values of pressure drop predicted from (10) has been found by trial to be less than ten percent.

The form of equation (9) should be quite satisfactory for describing the law of pneumatic conveying. However, as has been noted, this work revealed that the curves of Figures 3 to 5 are not precisely parallel to each other and that n varies somewhat. It was felt that a more general relation could be obtained from our data which would cover a greater variety of conditions than could be represented by an equation of the form of (9). Since it was early established that the pressure drop in the region of stable flow was essentially linear, it seemed advantageous to use the Fanning friction factor versus Reynolds number relation as employed in pipe friction work. Thus starting with the Fanning relation

$$f = \frac{2gDh}{u_a^2 L} \quad , \qquad (11)$$

where g_c is the dimensional constant, D is the pipe diameter, h the head loss in length L and u_a the air velocity, the value of h was modified so as to include the effect of the solids present. This was done as follows: Let W_a and W_s represent the weight of air and solids, respectively. Then is the volume of the solids as compared with that of the air flowing is neglected, there results a reasonably close approximation to the modified Fanning factor,

$$f_m = \frac{2g_cD}{u_a^2L}\,\frac{h}{1+R} = \frac{2g_cDh_m}{u_a^2L}, \tag{12}$$

where $R = W_s/W_a$ and h_m is written for the ratio $h/(1 + R)$.

It is possible to make the same modification in connection with Reynolds' number and obtain

$$(Re)_m = \frac{Du\rho_a(1 + R)}{\mu} = \frac{Du\rho_m}{\mu}; \tag{13}$$

where ρ_a is the density of the air and ρ_m is the mixture density based on the feed rate. No correction is required for μ since the assumption that the volume of the solids is negligible as compared with the volume of air has been made.

Equations (12) and (13) were developed and used by the author before he bacame aware of the fact that Lapple (9) had proposed such a relation as being a feasible one. If now one proceeds in accordance with usual practice and plots (12) versus (13), some sort of a functional relationship should be obtained. This was done with the data with results as shown in Figure 6. These results were at first disappointing since, as may be seen, the data do not lie on a single continuous curve, but distribute themselves by run in a series of more or less parallel lines and distribute themselves over a fairly wide band. Thus, the form of relations suspected as being the correct one by the author and Lapple (9) appears to fail.

However, it was not difficult to determine the reason for this failure and to modify the relation in a form which compensated for the effect described. As mentioned earlier, although the loading for a single run is kept constant, the varying air rate caused a change in the concentration of the solids. The data show that for a single run the modified Fanning factor remains relatively unchanged for a wide range of Reynolds' numbers. Some factor must, therefore, be introduced which sensibly affects the change in concentration during a run and thus (a) reduces the modified Reynolds' number to a single value, or (b), shifts the points constituting the separate runs so that they conform to some average curve drawn through the distribution of Reynolds' numbers. The latter seemed to be the simplest procedure, since any modification of Reynolds' number would cause it to lose its identity. Accordingly, the modified Fanning factor (12) was multiplied by a correction factor $(\rho_m/\rho_s)^{0.25}$ where ρ_s is the density of the solid. The exponent was obtained by trial and error until the points of each run rotated to form a single continuous curve. The data correlated by plotting

$$\frac{2g_c D\, h_m}{u_a^2\, L} \; (\rho_m/\rho_s)^{0.25} \quad \text{versus} \quad \frac{Du\rho_m}{\mu}$$

are shown in Figure 7. The correlation is seen to be quite good.

It needs to be pointed out that in the limit ρ_m/ρ_s should approach unity. That is, when no solids are flowing, the friction measured should be that of air alone. This can be taken care of by noting that we may write without affecting the accuracy of the relation

$$\frac{\rho_m}{\rho_s} \cong \frac{\rho_m}{\rho_s + \rho_a} ,$$

since the density of the air, ρ_a, is negligible as compared with the density of the solid. If we assume that if the term on the right is intended, then the equation

$$\frac{2g_c D\, h_m}{u_a^2\, L} \left(\rho_m/\rho_s + \rho_a\right)^{0.25} = f\,(\mathrm{Re})_m \tag{14}$$

conforms with the physical requirements of the pneumatic conveying problem.

<u>Generalization of Equation (14)</u>

Equation (14) applies to materials of specific gravity 1.0 in a three inch standard pipe. It applies to a wide variety of shapes and sizes. The correlation indicated in Figure 7 shows that these factors are not as important as some investigators have assumed them to be. However, it should be pointed out that one should not expect equation (14) to hold for pipes of small diameter, say less than one inch. The particles of large size in such a case tend to cut down the area of free flow and one might expect different conditions to be attained. Fortunately pneumatic conveying is rarely carried on in pipes of less than two inches diameter so that one might expect equation (14) to hold for the usual pipe sizes employed in pneumatic conveying, provided the same materials were used.

With materials of specific gravity greater than 1.0, the energy requirements to establish uniform motion in a horizontal pipe would be greater. Also, the pre-test section might have to be longer than that used in the experiments here described. Some tests made with spherical particles of porous alundum (sp. gr. 1.83) which averaged close to 3/8 inch in diameter indicate (a) that the friction factor versus Reynolds' number curve lies considerably higher than the one shown in Figure 7, and further, (b) that the material tends to concentrate toward the bottom of the pipe. This applies to moderate loadings (30 to 40 lb./min.) at Reynolds' numbers around 350,000. Under these conditions it would be expected that the exponent n in equation (9) would be higher, and following the experience of Howard (8) that it would be a function of the concentration.

Since these studies did not apply to material having a specific gravity much greater than unity, there are only a limited amount of data to indicate how equation (14) might be modified to correct for this material of higher specific gravity. It has been observed that the correction factor $[\rho_s/(\rho_a + \rho_s)]^{0.25}$ (or what has been used, $[\rho_s/\rho_a]^{0.25}$) is quite sensitive if the exponent is changed, It would seem, on the basis of the observations, that this exponent is definitely a function of the density of the material and that it would be in order to write equation (14) in the general form

$$[2g_c D\, h_m/u_a^2\, L][\rho_m/(\rho_s + \rho_a)]^{\Phi(\rho_s)} = f\ (Re)_m \quad . \qquad (15)$$

Further experiments would be required to determine the form of $\bar{\Phi}(o_s)$ to fit the data.

Studies on the Relative Velocity of Solids in Air

A series of high speed motion pictures were made on various specific loadings of Tenite to determine the relative velocity of the air to that of the solid. Figures 8 and 9 are representative prints from the particles in motion through the glass observation section. The lines across the field are spaced at one inch intervals so that the distance traveled by a particle may be measured. Through a calibration of the film speed the measurements permit the calculation of the particle velocity. No attempt has been made to carry this study to the extent of correlating the slip to loading, particle size, shape and other characteristics of the solid which might contribute, since the facilities available did not permit extensive use of this technique.

The calculated velocities of solid and air are tabulated in Table 3 below. Than an increase in the loading causes an increase in the relative velocity of the air is indicated. This idea is in accord with the experience in vertical settling, that is, the concept of the terminal velocity of a material is inherent and a decrease in terminal velocity with increased concentration of particles (hindered settling) is implied.

TABLE III

Material - Tenite

Feed	W_a	u_a	u_s	$u_a - u_s$	R	^{o}ds
Lb./Min.	Lb./Min.	Ft./Sec.	Ft./Sec.	Ft./Sec.	$\frac{\text{Wt. solid per unit time}}{\text{Wt. air per unit time}}$	$\frac{\text{Wt. dispersed solid}}{\text{Cu. Ft.}}$
14.7	20.6	97	64	33	0.71	0.075
14.0	15.1	70	47	23	0.93	0.097
39.4	19.3	90	62	28	2.04	0.208
38.8	17.3	82	42	40	2.23	0.302

Effect of Pipe Roughening

The effect of the intentional roughening of the test section by conveying abrasive materials is shown in Figure 10. The pressure drop versus air flow rate for two different feeds of cottonseed are shown. These two curves, when compared with those of Figure 5 for the relatively smooth pipe, show that pipe roughness contributes very markedly to the pressure drop encountered for a given loading of solid material. Further comparison of the two figures indicates that differences exceeding twenty-five percent have been encountered in the region of highest loading. It is also apparent that the contribution of the solid's flow to the increase in pressure drop with increased roughness is a factor much more predominant than that of the air. This conclusion is demonstrated by the very different slopes exhibited by the curves of Figure 10 and Figure 5. As previously mentioned, no investigation prior to this one appears to have recognized the magnitude of the importance of pipe roughness as a contributing factor to the flow properties of suspended material. That it should be a consideration, however, is obvious if we recognize the fact that the moving particles are constantly striking the wall of the pipe and changing direction. Any roughness would tend to decelerate the particles in proportion to the degree of roughness encountered by the particles. Thus, the energy required to reaccelerate the particles would be reflected in the pressure drop sustained. Future studies should, therefore, concern themselves with the need of conditioning of the pipe wall. Undoubtedly, in an actual pneumatic system, the condition of the pipe wall is stabilized rather quickly. But, if the system is to be used to convey some other material than that originally used, one cannot predict what might be expected with regard to the pressure drop sustained without recourse to experimentation after the pipe walls have been restabilized. This effect is not so important for soft materials as it is when one passes from the use of soft to hard to soft materials once more. The hard materials are those which markedly affect the roughness of the pipe wall.

These investigations suggest further that some improvement might be achieved by preconditioning the pipe surface by coating it, especially if soft materials are used. This might be done by passing through the system some plastic material such as Tenite. It has been noted that in a short time, the wall of the pipe becomes coated with a hard smooth layer. This layer is quite persistent and tends to reduce the roughness.

Curve of Critical or Settling Velocity

The conditions under which slugging begins are shown in Figures 3 to 5. However, it is possible to reduce the slugging conditions for all the data obtained to a single curve by using the critical points of the various horizontal series shown in Figure 6. This has been done in Figure 11, which is a plot of the modified Fanning factor f_m versus the modified Reynolds' number Re_m. Thus for the material having a specific gravity of approximately 1.0, it is at once possible to establish the conditions at which slugging begins. The necessary modification of the relation

$$\frac{2g_c D\, h_m}{u_a^2\, L} = f\,(Re)_m \quad , \tag{16}$$

which represents the slugging curve, in order to take care of materials having specific gravities greater or less than 1.0 remains to be investigated.

Consideration of Figure 11 reveals a remarkably good definition of the slugging conditions for the materials Tenite and soybeans, with a fair definition of the conditions for cottonseed. Since measurements of pressure drop over the test section were most difficult to make in the case of cottonseed the latter is not surprising.

The equation of the curve of Figure 11 is:

$$f = 4.47 \ Re^{-\frac{1}{2}} \quad , \tag{17}$$

and therefore equation (16) may be written in the general form as

$$\frac{2 g_c D \ h_m}{u_a^2 \ L} = 4.47 \ Re^{-\frac{1}{2}} \quad . \tag{18}$$

CONCLUSIONS

As a result of these investigations on pneumatic conveying, the following conclusions may be drawn:

(1) The test section must be preceded by an entrance section at least one hundred pipe diameters in length in order to permit the particles conveyed to attain a uniform velocity. The necessary entrance section may be longer for materials of specific gravity greater than 1.0.

(2) The pressure drop per unit length of pipe in which solids are conveyed can be expressed by an equation of the form

$$\frac{2g_c D\ h_m}{u_a^2\ L}\left(\frac{\rho_m}{\rho_s}\right)^{0.25} = f\ (Re)_m \tag{14}$$

for material having a specific gravity of 1.0. For materials having a specific gravity greater than 1.0 the following equation is suggested:

$$\frac{2g_c D\ h_m}{u_a^2\ L}\ (\rho_m/\rho_s)^{\Phi(\rho_s)} = f\ (Re)_m \tag{15}$$

The exact form of $\Phi(\rho_s)$ needs to be investigated.

(3) For a given feed rate, the relative velocity of the particles conveyed with respect to the velocity of the air is relatively constant once true conveying has been established.

(4) Pipe wall roughness has a marked effect on the pressure drop sustained when material flows in a pipe. Uniform and consistent test results depend on adequate preconditioning of the pipe wall.

(5) Initial settling or slugging in a pipe can be expressed by

an equation of the form

$$\frac{2g_c D\, h_m}{u_a^2\, L} = f\ (Re) = (Re)_m^{-\frac{1}{2}}$$

RECOMMENDATIONS FOR FUTURE STUDIES

The present investigation has been limited to the flow of solid material having a specific gravity of approximately 1.0 in a three inch nominal diameter pipe. Among the many factors which this study revealed as requiring further investigation, the following are important:

(1) Since the acceleration of the particles to a uniform velocity requires a rather high expenditure of energy, some work should be done on this phase of pneumatic conveying. Efforts should be directed toward determining methods of quickly accelerating the particles. Undoubtedly much data has been obtained in industry on the design of nozzles to attain this result but fundamental information such as the effect of varying the inlet velocity of the air stream with respect to the velocity in the pipe line should be determined. Undoubtedly, most difficulties encountered in commercial installations with regard to the clogging of the line are due to poor acceleration of the solid material. Since the clogging usually occurs near the entrance a common remedy is to design so that the air velocity in the line is much greater than is necessary. The inefficiency of such a design is obvious.

(2) The underlying causes of the slugging type of flow needs to be investigated. Effort should be directed toward a differentiation between the region of true conveying and the region of slugging. Also, the connection between the flow in the slugging region with type of flow encountered in the movement of fluidized solids should be of interest in view of the observations in this study and that of Farber (5). Since

at the slugging points, the pressure pattern changes, although conveying in a sense continues, an opportunity is offered to investigate when true conveying is actually taking place. It is possible that the intersection of the slugging curve with that of true conveying would offer a suitable criterion.

(3) Underlying the whole theory of pneumatic conveying is the fact that the relative velocity of the air with respect to the particle velocity has a most important bearing on the pressure drop. The use of the high speed camera offers a means of determining the true velocities of particles of different sizes and densities. When more information is available the correlation of slip velocity to the contributing variables should be capable of solution. It is certain that with this information a better generalization of the flow equation than developed in this thesis will be possible.

(4) The effect of particle density should be investigated in order to establish the functional dependence of the proposed correlating factor on the density.

(5) The effect of wall roughness might also offer a fruitful field of study, but would undoubtedly offer difficulties of solution due to the difficulty of maintaining other than an equilibrium degree of roughness for the harder materials. The effect of wall roughness is, however, too important a factor in pneumatic conveying to be neglected. Without proper characterization of roughness the generalization of the flow equation cannot be accomplished.

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APPENDIX I

FEEDER CALIBRATION

TABLE IV

FEEDER CALIBRATION DATA

1. Long Run - Feed Rate 21.2 Lb./Min. Tenite

Time, Sec.	Drum Weight, Lb.
0	14.0
45	29.6
90	45.4
135	61.5
180	77.3
225	93.0
270	109.0
315	125.1
360	140.8
405	156.7
450	172.6

2. Short Run - Feed Rate 52.0 Lb./Min Tenite.

Time, Sec.	Weight, Lb.
0	0
10.0	8.7
22.9	20.0
35.0	30.6
40.1	35.0
50.7	44.1
61.1	52.6
70.5	61.0
80.3	69.7
95.2	82.5
111.0	96.3
120.0	104.1

APPENDIX II

EXPERIMENTAL AND CALCULATED DATA

TABLE V

EXPERIMENTAL DATA

Run No.	10' Test Section: Press. Drop, Inch H_2O	10' Test Section: Gauge Press., Inch H_2O Vacuum	Orifice: Press. Drop, Inch H_2O	Orifice: Gauge Press., Inch H_2O Vacuum	Feed Rate, Lb./Min.	Barom. Press., mm. Hg	Room Temp., °F.
Air Calibration:							
A	1.07	7.2	16.53	12.6		738	70
B	0.99	6.7	15.09	11.6			
C	0.93	6.3	14.07	10.8			
D	0.86	5.8	13.03	10.2			
E	0.80	5.5	12.02	9.4			
F	0.75	5.0	11.02	8.7			
G	0.70	4.5	10.04	8.0			
H	0.63	4.2	9.00	7.2			
I	0.57	3.8	8.00	6.4			
J	0.50	3.3	6.97	5.7			
K	0.44	2.8	6.02	4.9			
L	0.36	2.4	4.90	4.0			
Soybeans and Air:							
19 A	1.28	10.6	15.08	15.6	25.8	733	65
B	1.25	10.6	14.84	15.5	26.2		
C	1.23	10.3	14.45	15.3	25.6		
D	1.20	10.1	13.92	14.8	25.8		
E	1.20	10.1	13.98	14.9	25.5		
F	1.15	9.5	12.95	13.5	25.5		
G	1.02	8.5	11.04	12.4	25.5		
H	0.93	7.8	11.66	10.3	25.3		
* I	0.81	6.9	10.02	9.8	25.1		
20 A	1.16	9.6	15.18	14.0	12.5	742	66
B	0.99	7.8	12.81	12.1	12.5		
C	0.93	7.4	11.65	11.2	12.5		
D	0.81	6.6	10.19	9.8	12.6		

*Runs on this and subsequent pages of this table thus indicated refer to the observed beginning of the slugging condition of flow.

Run No.	10' Test Section Press. Drop Inch H_2O	10' Test Section Gauge Press. Inch H_2O Vacuum	Orifice Press. Drop Inch H_2O	Orifice Gauge Press. Inch H_2O Vacuum	Feed Rate Lb./Min.	Barom. Press. mm. Hg	Room Temp. °F.
20 E	0.69	5.6	8.42	8.5	12.8		
F	0.56	4.4	6.11	6.5	12.4		
* G	0.53	4.2	5.65	6.2	12.7		
H	0.73	5.9	8.83	8.9	12.7		
I	0.87	6.6	10.66	10.3	12.6		
J	0.91	7.2	12.04	11.1	12.6		
K	1.09	8.2	14.10	12.8	12.7		
21 A	1.20	11.3	14.66	16.4	32.8	742	67
B	1.30	11.3	14.48	16.3	32.4		
C	1.29	11.2	14.16	16.0	32.6		
D	1.24	10.9	13.63	15.7	32.3		
E	1.20	10.4	12.67	14.8	32.5		
F	1.13	9.8	11.92	14.1	32.6		
G	1.05	9.3	10.78	13.2	32.8		
H	0.96	8.6	9.44	12.0	32.4		
I	0.90	8.2	8.69	11.4	32.5		
* J	0.85	7.8	7.85	10.7	32.5		
22 A	1.51	14.8	12.57	19.4	62.7	742	65
B	1.45	14.4	12.44	19.2	61.8		
C	1.45	14.4	12.32	19.1	62.1		
D	1.45	14.4	12.32	19.2	62.7		
E	1.43	14.3	12.10	18.8	62.0		
F	1.41	14.2	11.78	18.6	62.6		
G	1.39	14.0	11.46	18.3	62.8		
* H	1.33	13.0	10.37	17.2	63.0		
I	1.36	13.6	11.07	17.8	-		
Air Only:							
A	1.08	7.1	16.54	12.2		740	69
B	0.63		9.00	7.2			
Tenite (Coarse) and Air:							
23 A	1.20	9.4	16.00	14.6	12.0	740	69
B	1.17	9.2	15.44	14.4	12.1		

Run No.		10' Test Section: Press. Drop, Inch H_2O	10' Test Section: Gauge Press., Inch H_2O Vacuum	Orifice: Press. Drop, Inch H_2O	Orifice: Gauge Press., Inch H_2O Vacuum	Feed Rate, Lb./Min.	Barom. Press., mm. Hg	Room Temp., °F.
23	C	1.11	8.8	14.32	13.4	12.0		
	D	1.01	8.0	12.86	12.4	11.8		
	E	0.90	7.6	11.63	11.4	11.6		
	F	0.84	6.6	10.14	10.2	11.4		
	G	0.74	5.8	8.50	8.8	11.2		
	H	0.56	4.6	8.23	6.8	11.0		
*	I	0.50	4.0	7.00	5.8	11.0		
24	A	1.26	10.4	15.51	15.4	17.7	740	69
	B	1.23	9.8	15.07	15.0	17.5		
	C	1.17	9.4	13.97	14.2	17.5		
	D	1.08	8.8	12.56	13.0	17.2		
	E	1.01	8.2	11.32	12.1	17.4		
	F	0.90	7.4	9.89	10.8	17.4		
	G	0.80	6.6	8.23	9.5	17.1		
	H	0.72	5.9	7.16	8.6	17.1		
*	I	0.64	5.3	5.95	7.5	17.2		
25	A	1.47	15.2	12.93	19.8	55.2	743	70
	B	1.46	15.1	12.83	19.7	54.6		
	C	1.44	14.9	12.74	19.6	54.8		
	D	1.44	14.8	12.61	19.5	54.6		
	E	1.42	14.8	12.52	19.4	54.2		
	F	1.39	14.6	12.66	19.3	53.4		
	G	1.38	14.5	12.30	19.0	53.9		
	H	1.35	14.3	12.06	18.9	53.7		
	I	1.33	14.1	11.69	18.4	53.1		
	J	1.30	13.9	11.48	18.2	53.5		
	K	1.29	13.8	11.20	17.9	53.5		
*	L	1.27	13.5	10.91	17.7	53.1		
26	A	1.37	13.1	13.97	18.0	38.3	739	64
	B	1.34	12.8	13.46	15.7	38.3		
	C	1.27	12.2	12.61	16.7	38.3		
	D	1.20	11.5	11.39	15.6	38.2		
	E	1.07	10.3	9.79	13.9	37.7		
*	F	0.98	9.6	8.58	13.9	37.2		
*	G	0.84	8.9	8.66	11.5	37.3		
*	H	0.73	8.4	5.54	10.6	37.1		

	Run No.	10' Test Section Press. Drop	Gauge Press.	Orifice Press. Drop	Gauge Press.	Feed Rate	Barom. Press.	Room Temp.
		Inch H_2O	Inch H_2O Vacuum	Inch H_2O	Inch H_2O Vacuum	Lb./Min.	mm. Hg	°F.
	Tenite (Fine) and Air:							
	27 A	1.25	11.9	14.54	16.8	25.2	739	72
	B	1.23	11.4	14.32	16.6	25.3		
	C	1.20	11.4	13.78	16.2	25.1		
	D	1.17	11.0	13.11	15.5	24.8		
	E	1.07	10.2	11.81	14.4	24.2		
	F	0.96	9.1	10.16	12.7	24.0		
	G	0.92	8.7	9.39	12.0	23.5		
	H	0.87	8.2	8.66	11.4	23.5		
	I	0.83	8.0	7.96	11.0	24.2		
*	J	0.78	7.5	7.03	10.1	24.1		
*	K	0.65	6.4	5.56	8.8	23.5		
Def. Settling	L	0.58	6.6	4.59	8.5	23.9		
	Cottonseed and Air:							
	28 A	1.23	12.6	13.99	17.7	40.4	737	68
	B	1.22	12.5	13.87	17.6	40.8		
	C	1.20	12.4	13.56	17.3	40.8		
	D	1.17	12.2	12.99	16.9	41.3		
	E	1.11	11.5	12.10	16.1	41.1		
*	F	1.02	10.9	12.92	14.9	41.2		
	G	1.07	11.1	11.41	15.5	41.6		
*	H	0.99	11.0	10.46	14.4	41.1		
	29 A	1.20	10.7	15.28	15.7	22.5	737	68
	B	1.19	10.6	15.06	15.6	22.5		
	C	1.16	10.4	14.74	15.4	22.7		
	D	1.13	10.1	14.14	14.8	22.4		
	E	1.07	9.6	13.14	14.0	22.5		
	F	0.98	8.8	11.78	12.9	22.3		
	G	0.94	8.5	11.18	12.3	22.2		
	H	0.90	8.2	10.50	11.7	22.4		
*	I	0.85	7.7	9.69	11.2	22.2		

Run No.	10' Test Section Press. Drop Inch H_2O	Gauge Press. Inch H_2O Vacuum	Orifice Press. Drop Inch H_2O	Gauge Press. Inch H_2O Vacuum	Feed Rate Lb./Min	Barom. Press. mm. Hg	Room Temp. °F.
30 A	1.14	9.3	16.05	14.4	12.1	740	65
B	1.12	9.1	15.51	14.1	12.1		
C	1.10	8.8	14.92	13.8	12.1		
D	1.05	8.4	14.26	13.1	12.1		
E	0.97	7.8	13.02	12.2	11.9		
F	0.85	6.9	11.03	10.6	11.9		
* G	0.75	5.7	8.25	9.1	12.1		

Air Only:

Run No.	Press. Drop	Gauge Press.	Press. Drop	Gauge Press.	Feed Rate	Barom. Press.	Room Temp.
A	1.02	7.0	15.20	11.8		740	65
B	0.92	6.4	13.58	10.7			
C	0.85	5.8	12.22	9.8			
D	0.62	4.4	8.92	7.3			

Runs 31 through 34, Inclusive - Abrasive Materials Conveyed to Roughen Pipe.

Cottonseed and Air:

Run No.	Press. Drop	Gauge Press.	Press. Drop	Gauge Press.	Feed Rate	Barom. Press.	Room Temp.
35 A	1.30	11.0	14.56	15.9	25.7	729	78
B	1.32	11.8	14.07	16.5	33.5		
C	1.31	11.9	13.88	16.4	33.3		
D	1.30	11.8		16.3	33.9		
E	1.27	11.6	13.21	16.0	34.2		
F	1.22	11.1	12.54	15.4	33.4		
G	1.18	10.7	11.72	14.6	33.2		
H	1.12	10.3	10.76	13.9	33.8		
I	1.09	10.0	10.24	13.5	33.9		
J	1.06	9.7	9.58	13.1	34.5		
K	1.03	9.2	8.88	12.4	33.7		
L	0.99	8.9	8.29	11.9	33.8		
36 A	1.36	12.4	14.66	17.5	34.3	734	64
B	1.36	12.4	14.48	17.6	33.8		
C	1.16	10.8	11.56	15.2	33.4		

Run No.	10' Test Section Press. Drop Inch H_2O	10' Test Section Gauge Press. Inch H_2O Vacuum	Orifice Press. Drop Inch H_2O	Orifice Gauge Press. Inch H_2O Vacuum	Feed Rate Lb./Min.	Barom. Press. mm. Hg	Room Temp. °F.
37 A	1.40	13.0	13.97	18.2	38.7	734	64
B	1.43	13.7	13.46	18.9	44.2		
C	1.42	13.7	13.32	18.9	45.2		
D	1.41	13.5	13.07	18.6	44.5		
E	1.38	13.3	12.70	18.3	45.0		
F	1.36	13.1	12.22	18.0	45.6		
G	1.32	12.6	11.47	17.4	46.0		
H	1.28	12.3	10.71	16.8	46.0		
* I	1.24	11.6	9.72	15.8	45.8		
Air Only:							
A	1.13	8.2	15.84	13.4		732	80
B	0.98	7.3	13.49	11.6			
C	0.88	6.5	11.94	10.4			
D	0.80	5.8	10.69	9.4			
E	0.81	5.8	10.80	9.4			
F	0.73	5.4	9.69	8.6			
G	0.66	4.8	8.58	7.7			
H	0.58	4.0	7.28	6.5			
I	0.49	3.4	5.80	5.4			
J	0.41	2.9	4.83	4.5			
Alundum Spheres and Air:							
38 A	1.510	10.0	15.48	15.3	15.1	736	68
B	1.510	9.8	15.26	15.1	15.3		
C	1.47	9.8	14.52	14.7	15.1		
D	1.44	9.3	13.75	14.6	15.0		
E	1.34	9.1	12.30	13.1	15.2		
F	1.31	8.1	11.10	13.2	14.9		
G	1.26	7.6	9.58	11.0	14.5		
# H	1.31	7.3	8.79	10.9	15.0		
# I	1.16	7.1	7.66	10.1	14.6		

#Glass observation section not used. Beginning of the slugging type of flow was estimated by the sound of the movement of material.

Run No.	10' Test Section Press. Drop	Gauge Press.	Orifice Press. Drop	Gauge Press.	Feed Rate	Barom Press.	Room Temp.
	Inch H_2O	Inch H_2O Vacuum	Inch H_2O	Inch H_2O Vacuum	Lb./Min.	mm. Hg	°F.
39 A	1.95	13.8	13.29	18.6	34.3	736	68
B	1.93	13.5	13.06	18.6	34.6		
C	1.91	13.0	12.65	17.1	34.4		
# D	1.98	12.9	11.92	17.3	34.3		
E	1.94	13.0	12.03	17.8	34.7		
40 A	1.93	12.6	14.17	17.4	25.6	738	68
B	1.90	12.0	13.69	17.0	25.1		
C	1.80	11.4	12.60	16.2	25.3		
D	1.76	10.4	10.20	14.2	25.3		
41 A	2.16	13.4	13.07	18.2	-	739	75
B	2.05	13.6	12.99	18.4	36.8		
C	1.96	13.8	12.81	18.8	36.8		
D	1.98	13.8	12.70	18.6	36.2		
E	2.00	13.4	12.37	18.6	36.3		
# F	2.00	12.8	12.56	18.0	33.2		
# G	1.87	13.0	12.01	17.6	34.5		

TABLE VI

CALCULATED DATA

Run No.	Press. Drop Test Sect. Inch H_2O	Air Rate: Weight Lb./Min.	Air Rate: Volumetric Ft^3/Min.	Density: Air Lb./Ft^3	Density: Mixture Lb./Ft^3	Head Loss L = 100 Ft. Ft-Lb_F/Lb_M	Re $\times 10^{-3}$	Friction Factor $\times 10^3$	$f(\rho_m/\rho_s)^{0.25}$ $\times 10^3$
Air Only - Calibration:									
A	1.07	21.05	294	0.0716		775	143	15.0	
B	0.99	20.17	282	.0716		717	137	15.1	
C	0.93	19.47	272	.0718		671	132	15.2	
D	0.86	18.80	262	.0718		621	127	15.1	
E	0.80	18.07	252	.0718		578	123	15.2	
F	0.75	17.35	242	.0719		541	118	15.4	
G	0.70	16.57	230	.0720		504	112	15.8	
H	0.63	15.73	218	.0721		453	107	15.9	
I	0.57	14.88	206	.0722		409	101	16.1	
J	0.50	13.90	192	.0723		359	94	16.3	
K	0.44	12.97	179	.0724		316	88	16.5	
L	0.36	11.73	162	.0725		258	80	15.8	

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate		Density		Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
		Weight	Volumetric	Air	Mixture				
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	Ft-Lb_F/Lb_M	x 10^{-3}	x 10^3	x 10^3
Soybeans and Air:									
19 A	1.28	20.0	282	0.0710	0.163	403	311	8.50	1.85
B	1.26	19.8	279	.0710	.165	394	312	8.55	1.86
C	1.23	19.6	275	.0712	.164	388	306	8.56	1.87
D	1.20	19.3	266	.0712	.168	368	303	8.69	1.90
E	1.20	19.3	266	.0712	.168	368	303	8.69	1.90
F	1.15	18.7	262	.0712	.169	351	300	8.53	1.88
G	1.02	17.3	242	.0715	.177	296	290	8.44	1.88
H	0.94	16.3	227	.0716	.183	262	281	8.49	1.90
* I	0.82	14.8	207	.0718	.195	216	274	8.42	1.91

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect. Inch H_2O	Air Rate: Weight Lb./Min.	Air Rate: Volumetric Ft^3/Min.	Density: Air Lb./Ft^3	Density: Mixture Lb./Ft^3	Head Loss L = 100 Ft. Ft-Lb_F/Lb_M	Re $\times 10^{-3}$	Friction Factor $\times 10^3$	$f(\rho_m/\rho_s)^{0.25}$ $\times 10^3$
Soybeans and Air:									
20 A	1.16	20.2	290	0.0718	0.115	525	226	10.4	2.07
B	0.99	18.6	257	.0724	.121	426	211	10.8	2.18
C	0.94	17.8	246	.0725	.123	388	205	10.7	2.17
D	0.81	16.7	230	.0725	.127	328	198	10.4	2.12
E	0.69	15.2	210	.0727	.134	269	191	10.2	2.11
F	0.56	13.1	179	.0731	.142	207	172	10.8	2.27
* G	0.53	12.6	172	.0731	.147	187	171	10.6	2.25
H	0.73	15.7	216	.0727	.131	293	192	10.5	2.16
I	0.87	17.1	236	.0725	.126	359	202	10.8	2.20
J	0.91	18.1	249	.0725	.123	387	208	10.4	2.11
K	1.09	19.5	269	.0724	.120	475	219	10.9	2.19

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate		Density		Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
		Weight	Volumetric	Air	Mixture				
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	Ft-Lb_F/Lb_M	$\times 10^{-3}$	$\times 10^3$	$\times 10^3$
Soybeans and Air:									
21 A	1.20	19.9	277	0.0718	0.190	326	357	7.10	1.61
B	1.30	19.8	275	.0718	.190	354	354	7.82	1.78
C	1.29	19.5	272	.0718	.192	347	354	7.84	1.77
D	1.25	19.1	267	.0718	.193	322	349	7.84	1.78
E	1.20	18.6	258	.0720	.199	313	348	7.85	1.80
F	1.13	18.0	250	.0720	.202	292	342	7.80	1.79
G	1.06	17.2	238	.0722	.210	260	339	7.67	1.80
H	0.96	16.0	223	.0722	.217	229	328	7.69	1.79
I	0.90	15.5	217	.0724	.222	212	326	7.62	1.77
* J	0.85	14.7	204	.0724	.233	187	322	7.50	1.78

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate		Density		Head Loss $L = 100$ Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
		Weight	Volumetric	Air	Mixture				
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	Ft-Lb_F/Lb_M	$\times 10^{-3}$	$\times 10^3$	$\times 10^3$
Soybeans and Air:									
22 A	1.51	18.5	260	0.0712	0.312	251	550	6.20	1.59
B	1.45	18.4	258	.0713	.310	243	542	6.09	1.55
C	1.45	18.2	255	.0713	.314	239	543	6.14	1.57
D	1.45	18.2	255	.0713	.314	239	543	6.14	1.57
E	1.43	18.1	254	.0713	.315	235	542	6.08	1.56
F	1.41	17.8	250	.0713	.321	228	543	6.09	1.56
G	1.39	17.6	249	.0713	.325	222	544	6.07	1.57
* H	1.34	16.8	236	.0715	.338	206	541	6.17	1.61
I	1.36	17.3	242	.0713	Feed Rate Not Recorded		-	-	-

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate Weight	Air Rate Volumetric	Density Air	Density Mixture	Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	$Ft\text{-}Lb_F/Lb_M$	$\times 10^{-3}$	$\times 10^3$	$\times 10^3$
Tenite (Coarse) and Air:									
23 A	1.20	20.7	289	0.0715	0.113	552	221	11.0	2.20
B	1.17	20.3	284	.0715	.114	536	219	11.1	2.23
C	1.11	19.6	274	.0715	.115	505	212	11.2	2.25
D	1.02	18.6	260	.0716	.117	459	206	11.4	2.30
E	0.90	17.7	247	.0717	.119	392	199	10.7	2.18
F	0.84	16.6	232	.0718	.121	363	190	11.3	2.30
G	0.74	15.3	213	.0720	.125	307	181	11.3	2.32
H	0.56	13.2	184	.0721	.132	222	165	11.4	2.38
* I	0.50	12.0	166	.0723	.139	189	156	11.5	2.43

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate		Density		Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
		Weight	Volumetric	Air	Mixture				
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	$Ft\text{-}Lb_F/Lb_M$	$\times 10^{-3}$	$\times 10^3$	$\times 10^3$
Tenite (Coarse) and Air:									
24 A	1.26	20.3	285	0.0713	0.133	494	257	10.2	2.12
B	1.23	20.1	282	.0713	.133	478	254	10.0	2.08
C	1.17	19.3	271	.0714	.136	444	250	10.1	2.12
D	1.08	18.4	258	.0715	.138	405	241	10.2	2.15
E	1.01	17.5	244	.0717	.143	368	236	10.3	2.18
F	0.90	16.4	228	.0718	.147	319	227	10.4	2.22
G	0.80	15.1	210	.0719	.153	272	218	10.3	2.22
H	0.72	14.1	196	.0720	.159	245	211	10.6	2.32
* I	0.64	13.0	180	.0722	.167	196	204	10.1	2.24

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate Weight	Air Rate Volumetric	Density Air	Density Mixture	Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	Ft-Lb_F/Lb_M	x 10^{-3}	x 10^3	x 10^3
Tenite (Coarse) and Air:									
25 A	1.47	18.7	265	0.0704	0.278	277	500	6.82	1.71
B	1.46	18.6	264	.0704	.277	274	496	6.56	1.65
C	1.44	18.5	263	.0704	.278	269	496	6.50	1.63
D	1.44	18.4	262	.0705	.278	269	493	6.55	1.64
E	1.42	18.3	260	.0705	.279	264	492	6.43	1.61
F	1.39	18.4	262	.0706	.275	263	488	6.40	1.60
G	1.38	18.2	258	.0707	.279	256	488	6.42	1.61
H	1.35	18.0	255	.0707	.281	249	486	6.40	1.61
I	1.33	17.8	252	.0708	.281	246	480	6.47	1.63
J	1.30	17.7	250	.0708	.284	238	482	6.36	1.60
K	1.29	17.5	247	.0709	.287	235	481	6.43	1.63
* L	1.27	17.3	243	.0709	.289	229	476	6.48	1.64

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate: Weight	Air Rate: Volumetric	Density: Air	Density: Mixture	Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	$Ft\text{-}Lb_F/Lb_M$	$\times 10^{-3}$	$\times 10^3$	$\times 10^3$
26 A	1.37	19.2	268	0.0715	0.210	339	381	7.88	1.84
B	1.34	18.8	263	.0715	.217	320	387	7.73	1.82
C	1.27	18.3	256	.0717	.221	299	383	7.62	1.81
D	1.20	17.5	244	.0717	.228	272	377	7.63	1.82
E	1.07	16.3	226	.0720	.238	234	365	7.65	1.84
* F	0.98	15.2	212	.0720	.246	208	353	7.72	1.88
* G	0.84	13.5	187	.0722	.271	162	343	7.73	1.93
* H	0.73	12.3	170	.0724	.289	131	333	7.57	1.92

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect. Inch H_2O	Air Rate: Weight Lb./Min.	Air Rate: Volumetric Ft^3/Min.	Density: Air Lb./Ft^3	Density: Mixture Lb./Ft^3	Head Loss L = 100 Ft. Ft-Lb_F/Lb_M	Re $\times 10^{-3}$	Friction Factor $\times 10^3$	$f(\rho_m/\rho_s)^{0.25}$ $\times 10^3$
Tenite (Fine) and Air:									
27 A	1.25	19.7	278	0.0706	0.161	403	303	8.71	1.90
B	1.23	19.5	276	.0707	.162	392	303	8.59	1.88
C	1.20	19.2	271	.0708	.163	380	299	8.64	1.90
D	1.17	18.8	266	.0709	.164	369	296	8.71	1.92
E	1.07	17.8	251	.0710	.167	332	284	8.80	1.95
F	0.96	16.5	232	.0712	.174	286	274	8.87	1.98
G	0.92	15.8	222	.0712	.177	269	266	9.12	2.04
H	0.87	15.4	216	.0713	.180	250	264	8.95	2.05
I	0.83	14.7	206	.0713	.188	227	262	8.93	2.03
J	0.78	14.0	196	.0714	.194	207	258	8.93	2.04
K	0.65	12.6	174	.0715	.206	164	243	9.03	2.10
L	0.58	11.3	158	.0714	.222	136	238	9.10	2.16

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate		Density		Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
		Weight	Volumetric	Air	Mixture				
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	$Ft\text{-}Lb_F/Lb_M$	x 10^{-3}	x 10^3	x 10^3
28 A	1.23	19.2	274	0.0707	0.218	292	405	6.42	1.56
B	1.22	19.1	272	.0707	.221	286	407	6.45	1.57
C	1.20	19.0	269	.0709	.223	279	406	6.43	1.57
D	1.17	18.7	264	.0709	.229	265	410	6.35	1.56
E	1.11	18.0	254	.0709	.233	247	401	6.38	1.58
F	1.02	17.2	241	.0711	.242	219	395	6.29	1.56
G	1.07	17.7	246	.0711	.240	234	400	6.45	1.60
* H	0.99	16.8	237	.0711	.245	210	393	6.23	1.56

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate Weight	Air Rate Volumetric	Density Air	Density Mixture	Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	$Ft\text{-}Lb_F/Lb_M$	$\times 10^{-3}$	$\times 10^3$	$\times 10^3$
29 A	1.20	20.2	284	0.0711	0.150	414	289	8.56	1.89
B	1.19	20.0	282	.0711	.151	408	288	8.58	1.91
C	1.16	19.8	278	.0713	.153	395	288	8.54	1.90
D	1.13	19.5	274	.0713	.153	382	284	8.50	1.90
E	1.07	18.8	265	.0713	.155	362	278	8.52	1.90
F	0.98	17.8	249	.0715	.161	316	272	8.51	1.92
G	0.94	17.3	242	.0715	.163	298	267	8.50	1.92
H	0.90	16.8	234	.0717	.168	276	267	8.42	1.92
* I	0.85	16.3	228	.0717	.169	259	261	8.38	1.92

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate		Density		Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
		Weight	Volumetric	Air	Mixture				
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	$Ft\text{-}Lb_F/Lb_M$	$x\ 10^{-3}$	$x\ 10^3$	$x\ 10^3$
30 A	1.14	20.7	286	0.0721	0.114	519	221	10.5	2.18
B	1.12	20.3	282	.0721	.115	505	210	10.6	2.20
C	1.10	20.0	277	.0721	.116	495	218	10.7	2.24
D	1.05	19.5	270	.0723	.117	466	196	10.7	2.22
E	0.97	18.7	259	.0723	.118	428	189	10.7	2.22
F	0.85	17.3	239	.0725	.122	366	198	10.7	2.24
* G	0.75	15.1	208	.0727	.133	292	187	11.3	2.42
Air Only:									
A	1.02	20.34							
B	0.90	19.25							
C	0.85	18.35							
D	0.63	15.67							

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate		Density		Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
		Weight	Volumetric	Air	Mixture				
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	$Ft\text{-}Lb_F/Lb_M$	$\times 10^{-3}$	$\times 10^3$	$\times 10^3$
Runs 31 through 34, Pipe Roughened by Abrasive Materials. Data Not Recorded.									
Cottonseed and Air:									
35 A	1.30	Bad Reading							
B	1.32	19.3							
C	1.31	19.3							
D	1.30	-							
E	1.27	18.7							
F	1.22	18.3							
G	1.19	17.8							
H	1.13	17.1							
I	1.09	16.7							
J	1.06	16.2							
K	1.03	15.7							
L	0.99	15.1							

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate		Density		Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
		Weight	Volumetric	Air	Mixture				
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	Ft-Lb_F/Lb_M	$\times 10^{-3}$	$\times 10^3$	$\times 10^3$
Run 36 Interrupted.									
37 A	1.40	-							
B	1.43	19.0							
C	1.42	18.8							
D	1.41	18.7							
E	1.38	18.6							
F	1.36	18.1							
G	1.32	17.5							
H	1.28	17.0							
I	1.24	16.3							

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate		Density		Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
		Weight	Volumetric	Air	Mixture				
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	Ft-Lb_F/Lb_M	x 10^{-3}	x 10^3	x 10^3
Air Only:									
A	1.13	20.3	291	0.0698		836	138	16.5	
B	0.98	18.8	278	.0700					
C	0.88	17.8	254	.0701					
D	0.78	16.9	240	.0702					
E	0.81	17.0	242	.0702					
F	0.73	16.1	239	.0704					
G	0.65	15.2	215	.0705		483	103	17.4	
H	0.58	14.0	198	.0706					
I	0.48	12.6	178	.0707					
J	0.41	11.5	163	.0708					

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate Weight	Air Rate Volumetric	Density Air	Density Mixture	Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	$Ft\text{-}Lb_F/Lb_M$	$x\ 10^{-3}$	$x\ 10^3$	$x\ 10^3$
Alundum Spheres and Air:									
38 A	1.51	20.4	286	0.0711	0.123	641	239	13.6	2.47
B	1.51	20.2	284	.0711	.125	626	241	13.0	2.37
C	1.47	19.7	277	.0711	.125	613	235	13.4	2.44
D	1.44	19.2	269	.0713	.127	590	231	13.5	2.47
E	1.34	18.2	255	.0713	.131	534	227	13.7	2.52
F	1.31	17.3	242	.0715	.133	516	218	14.7	2.72
G	1.26	16.3	226	.0715	.136	483	208	15.8	2.94
# H	1.31	15.5	217	.0715	.141	475	207	16.8	3.15
# I	1.16	14.5	202	.0717	.144	418	197	17.1	3.23

#Glass observation section not used. Beginning of the slugging type of flow was estimated by the sound of the movement of material.

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate Weight	Air Rate Volumetric	Density Air	Density Mixture	Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb/Ft^3	Lb./Ft^3	Ft-Lb_F/Lb_M	x 10^{-3}	x 10^3	x 10^3
39 A	1.95	18.7	267	0.0703	0.199	507	360	11.9	2.43
B	1.93	18.5	264	.0703	.201	498	360	11.9	2.44
C	1.91	18.3	260	.0705	.203	479	358	11.8	2.42
# D	1.98	17.8	252	.0705	.206	496	352	13.1	2.70
E	1.94	17.9	253	.0705	.207	486	355	12.7	2.63
40 A	1.93	19.4							
B	1.90	19.1							
C	1.80	18.3							
D	1.76	16.5							

Further quantities on spheres not calculated in view of the uncertainty as to the flow condition as is mentioned in the text.

TABLE VI (Continued)

CALCULATED DATA

Run No.	Press. Drop Test Sect.	Air Rate		Density		Head Loss L = 100 Ft.	Re	Friction Factor	$f(\rho_m/\rho_s)^{0.25}$
		Weight	Volumetric	Air	Mixture				
	Inch H_2O	Lb./Min.	Ft^3/Min.	Lb./Ft^3	Lb./Ft^3	Ft-Lb_F/Lb_M	x 10^{-3}	x 10^3	x 10^3
41 A	2.16	18.7							
B	2.05	18.5							
C	1.96	18.4							
D	1.98	18.3							
E	2.00	18.0							
# F	2.00	18.3							
# G	1.87	17.9							

TABLE VII

DATA FOR EQUILIBRIUM STUDY

Feed	Length of Test Sect.	Pressure Drop	Pressure Drop	Average Press. Drop / Unit Length
Lb./Min.	Ft.	Inch H_2O	Inch H_2O	Inch H_2O/Ft.
19.0	45	8.540	8.540	0.190
	40	5.568	5.560	0.139
	35	4.720	4.710	0.135
19.1	30	3.965	3.955	0.132
	25	3.310	3.300	0.132
19.0	20	2.620	2.620	0.131
53.6	45	9.100	9.050	0.202
	40	6.923	6.920	0.173
	35	5.470	5.480	0.156
54.0	30	4.485	4.475	0.150
	25	3.675	3.670	0.147
53.8	20	2.910	2.912	0.145

APPENDIX III

SAMPLE CALCULATION

SAMPLE CALCULATION

The method of calculation of the values listed in Table VI will be illustrated for soybeans with an air flow rate of 19.8 Lb./Min., calculated from the orifice reading using the equation and coefficients presented by Stearns, et al. (10).

$D = 3.108$ in.

$\rho_s = 73.0$ lb./ft^3

At 1.30 in. H_2O pressure drop across the test section,

$W_s = 32.4$ lb./min. = feed rate

$\rho_a = 0.0718$ lb./ft^3

$\mu = 0.018$ centipoise = viscosity of air

$Q_a = W_a/\rho_a = 19.8/.0718 = 275$ ft^3/min.

$$\rho_m = \frac{W_s + W_a}{Q_s + Q_a} = \frac{\frac{W_s}{Q_a} + \rho_a}{\frac{W_s}{\rho_s Q_a} + 1} = \frac{\frac{W_s}{Q_a} + \rho_a}{1} = 0.190 \text{ lb./ft}^3$$

$$h_m = h_w \times (\rho_w/\rho_m) = 1.30 \times 10/12 \times 62.3/0.190 = 354 \frac{\text{Ft-Lb}_F}{\text{Lb}_M}$$

$L = 100$

$$Re = \frac{D\,Q\,\rho}{\mu\,A} = \frac{3.108/12 \times 275 \times 0.190 \times 144}{.018 \times 6.72 \times 10^{-4} \times .7854 \times (3.108)^2 \times 60} =$$

$$= 354{,}000$$

$$f = 2g_c D\, h_m/u_a^2\, L = \frac{2 \times 32.2 \times \pi^2 \times (3.108)^5 \times 354 \times 3600}{16 \times 12^5 \times 100 \times 275^2} = .00782$$

$$f\,(\rho_m/\rho_s)^{0.25} = (.00782)(0.190/73.0)^{0.25} = 0.00177$$

APPENDIX IV

FIGURES

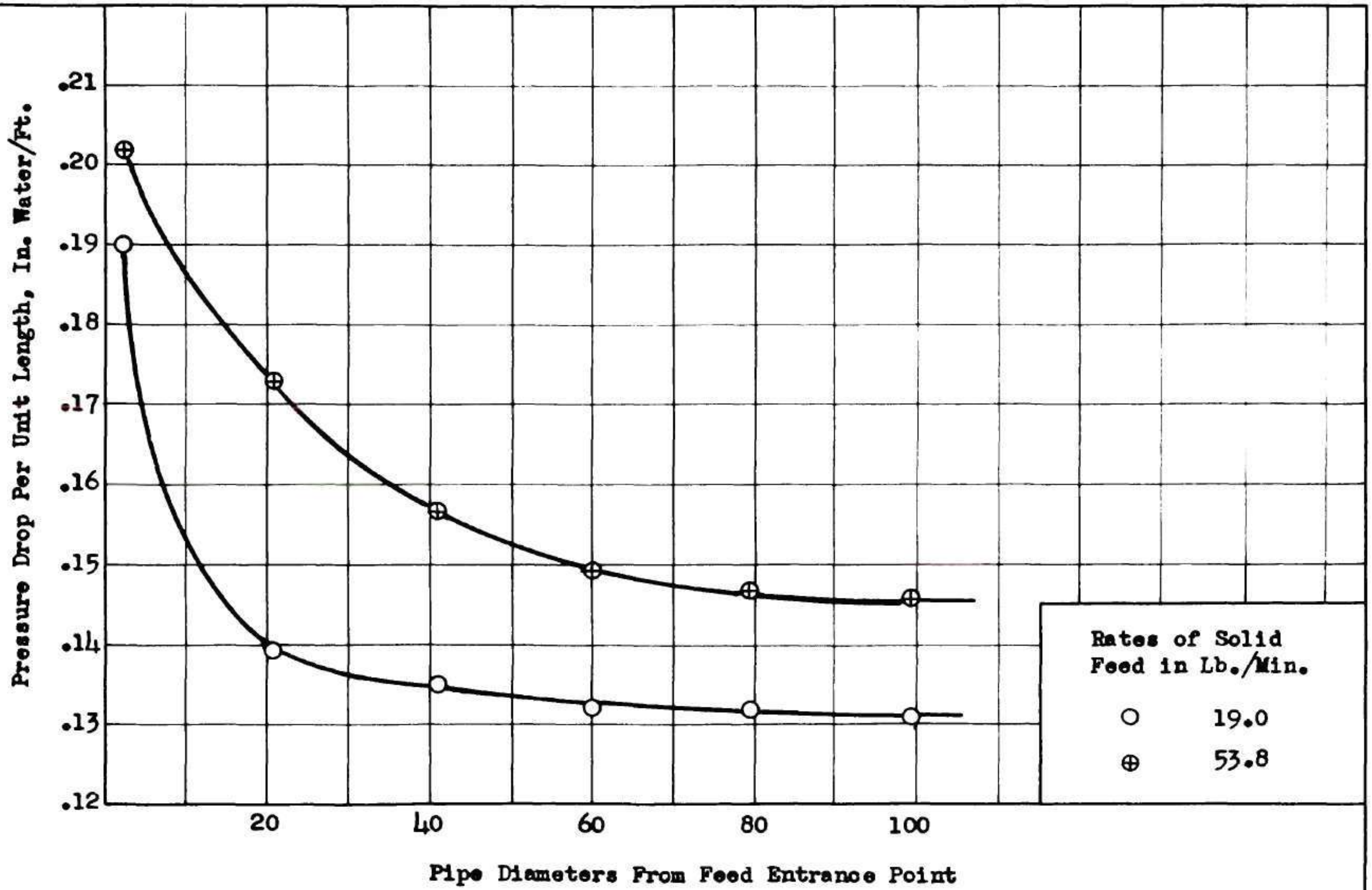

Figure 2. Average Pressure Gradient Along the Conveying Line for Two Tenite Feed Rates at Constant Air Flow Rates.

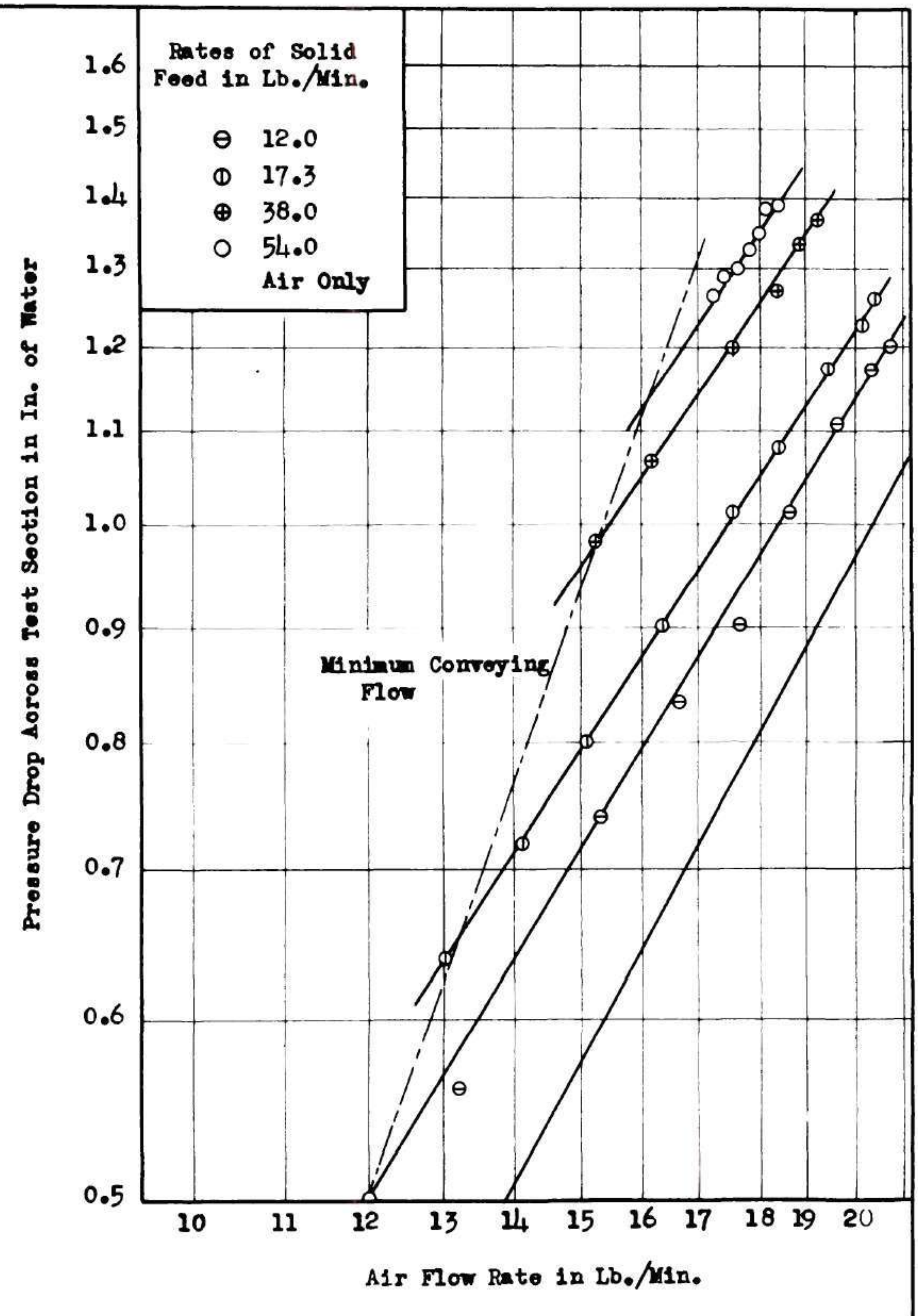

Figure 3. Pressure Drop - Air Flow Relationship for Tenite.

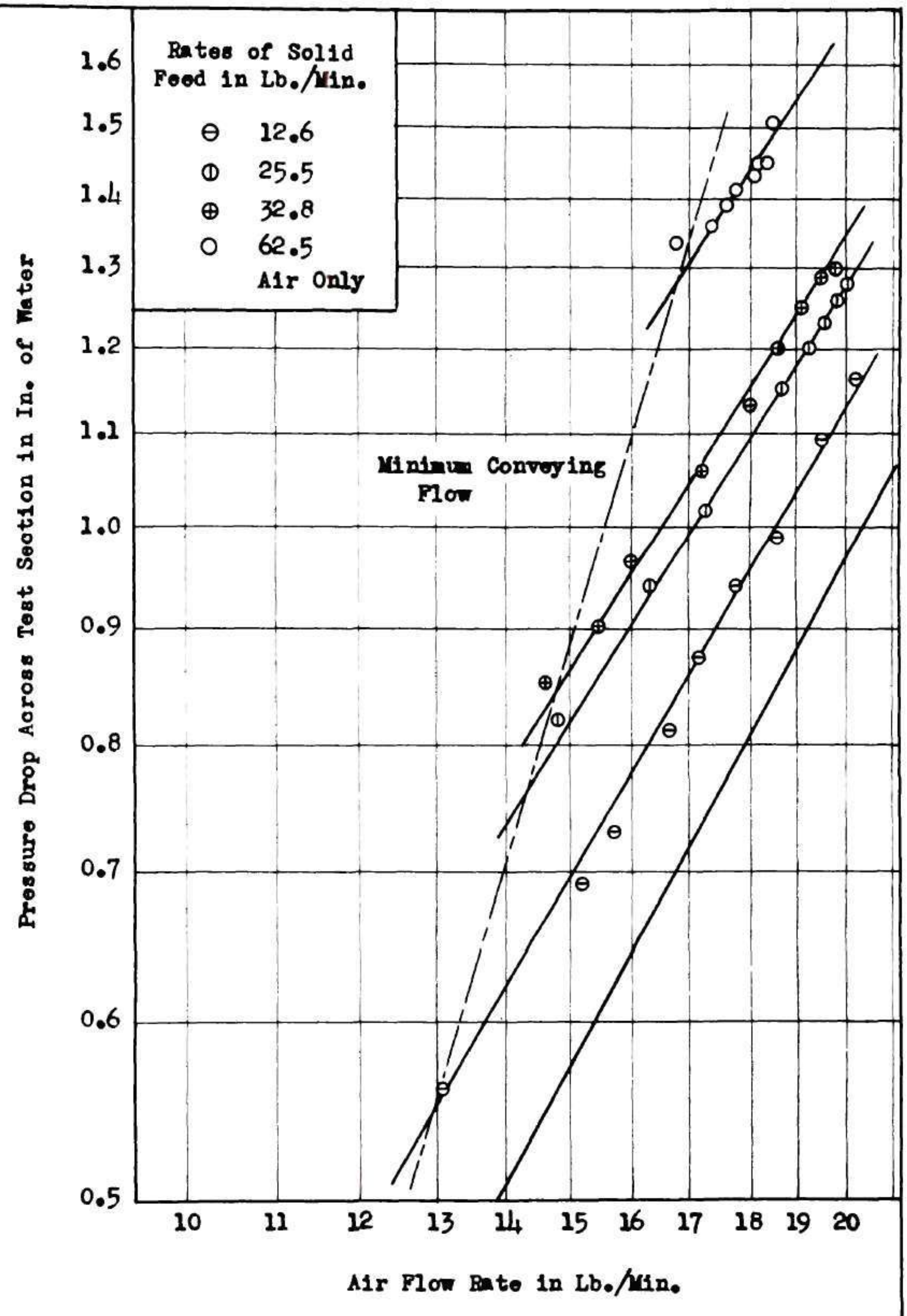

Figure 4. Pressure Drop - Air Flow Relationship for Soybeans.

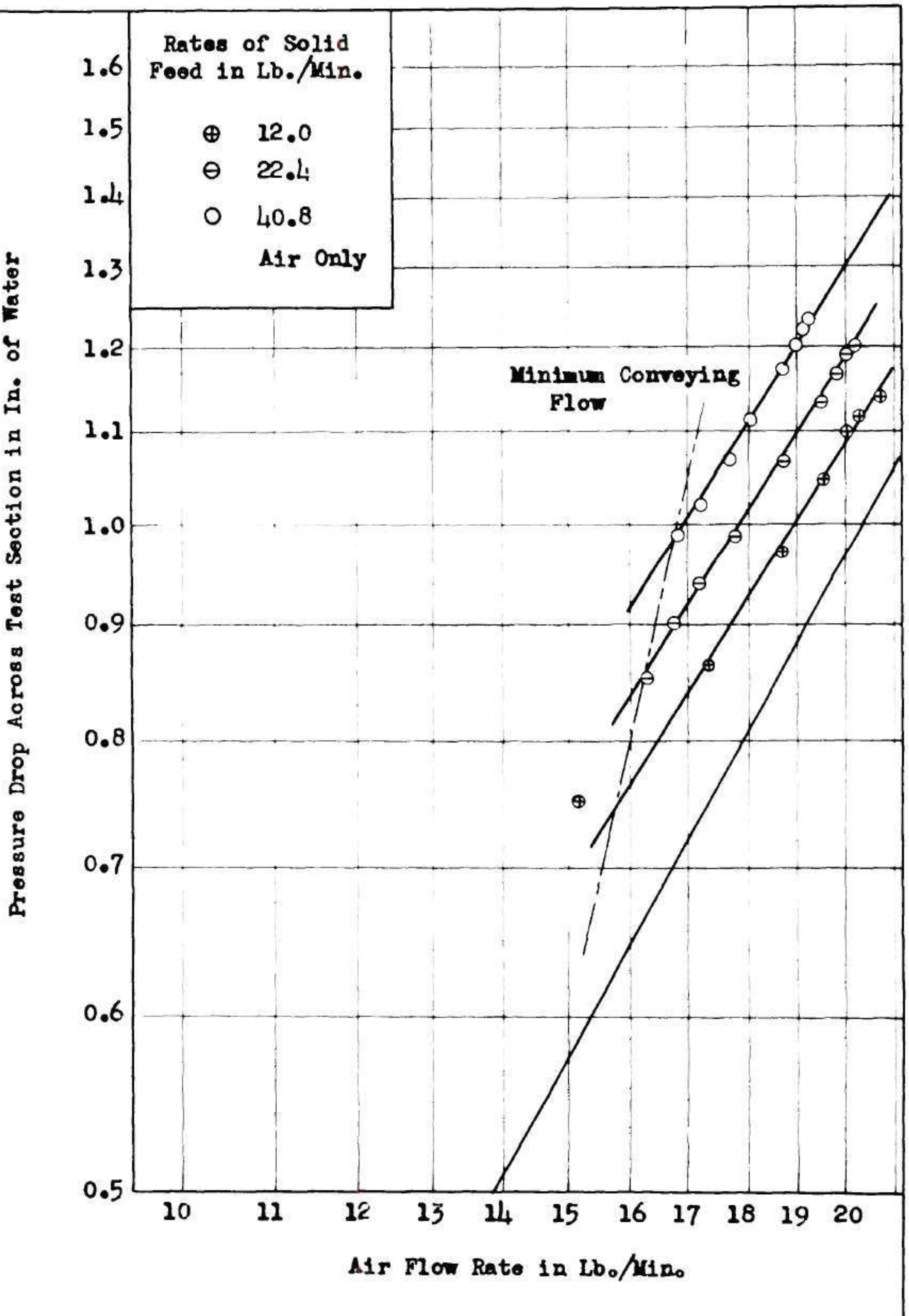

Figure 5. Pressure Drop - Air Flow Relationship for Cottonseed.

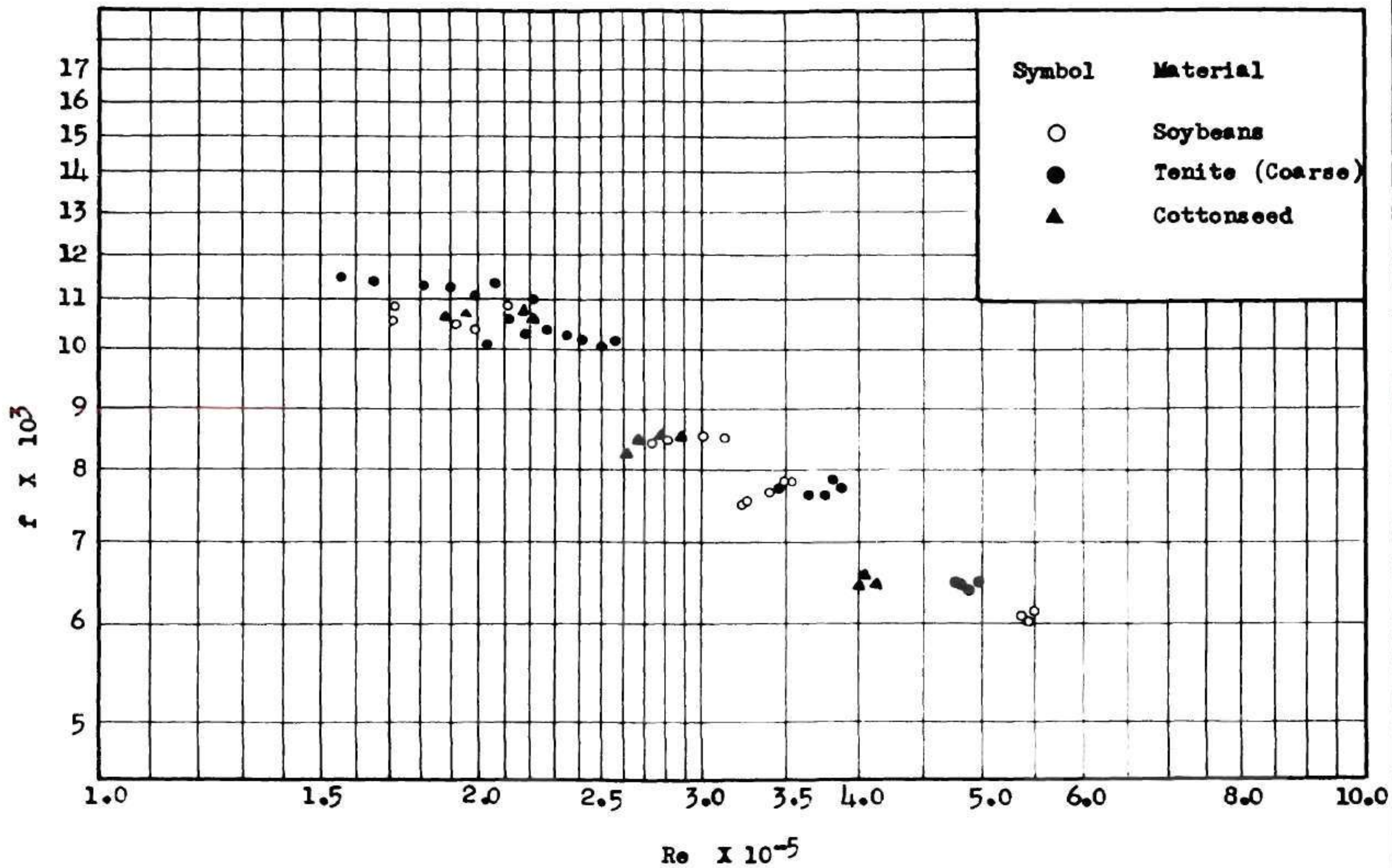

Figure 6. Modified Friction Factor and Reynolds Number Relationship for Soybeans, Tenite, and Cottonseed.

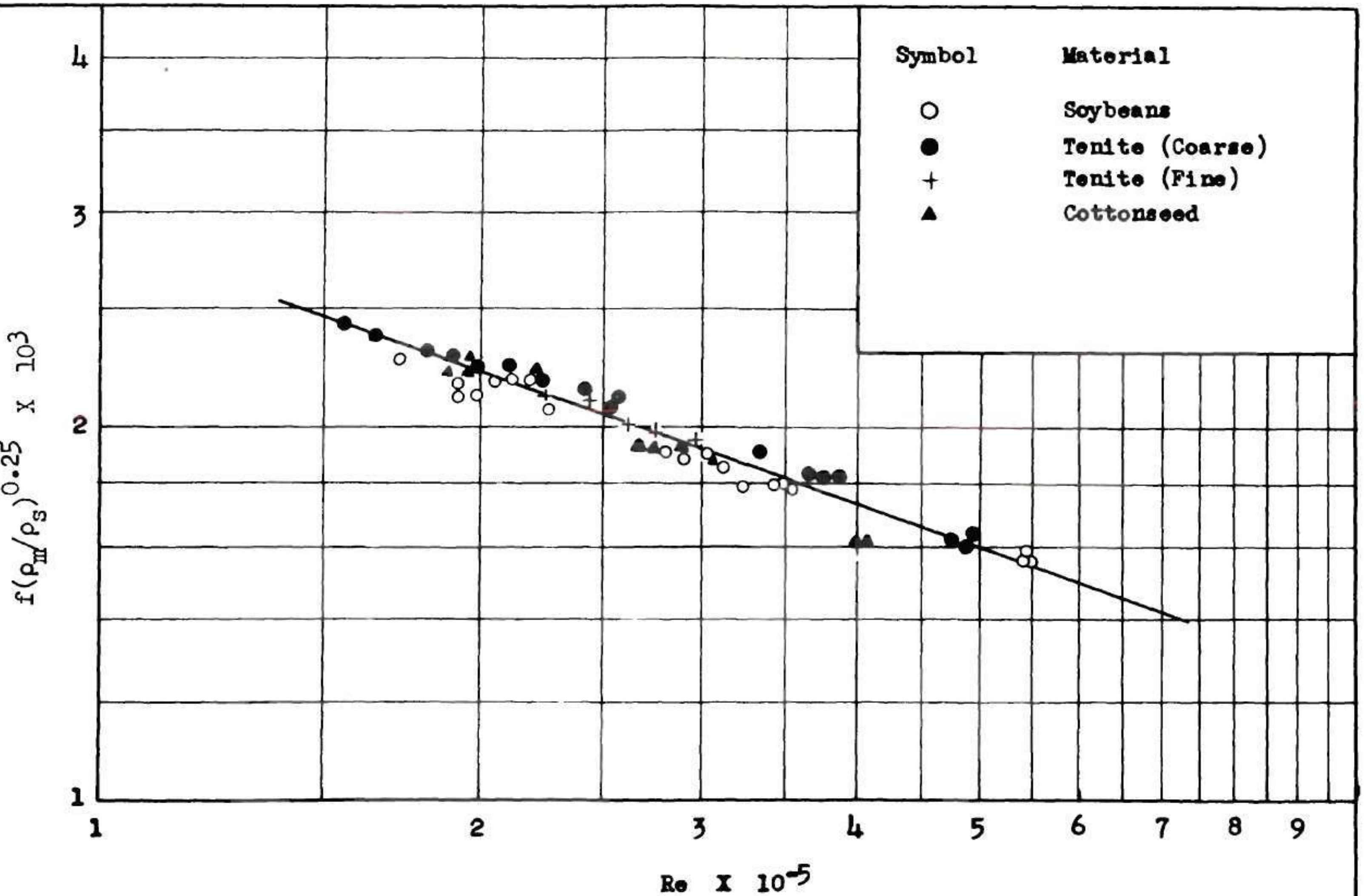

Figure 7. Modified Friction Factor and Reynolds Number Relationship With the Correlating Factor $(\rho_m/\rho_s)^{0.25}$

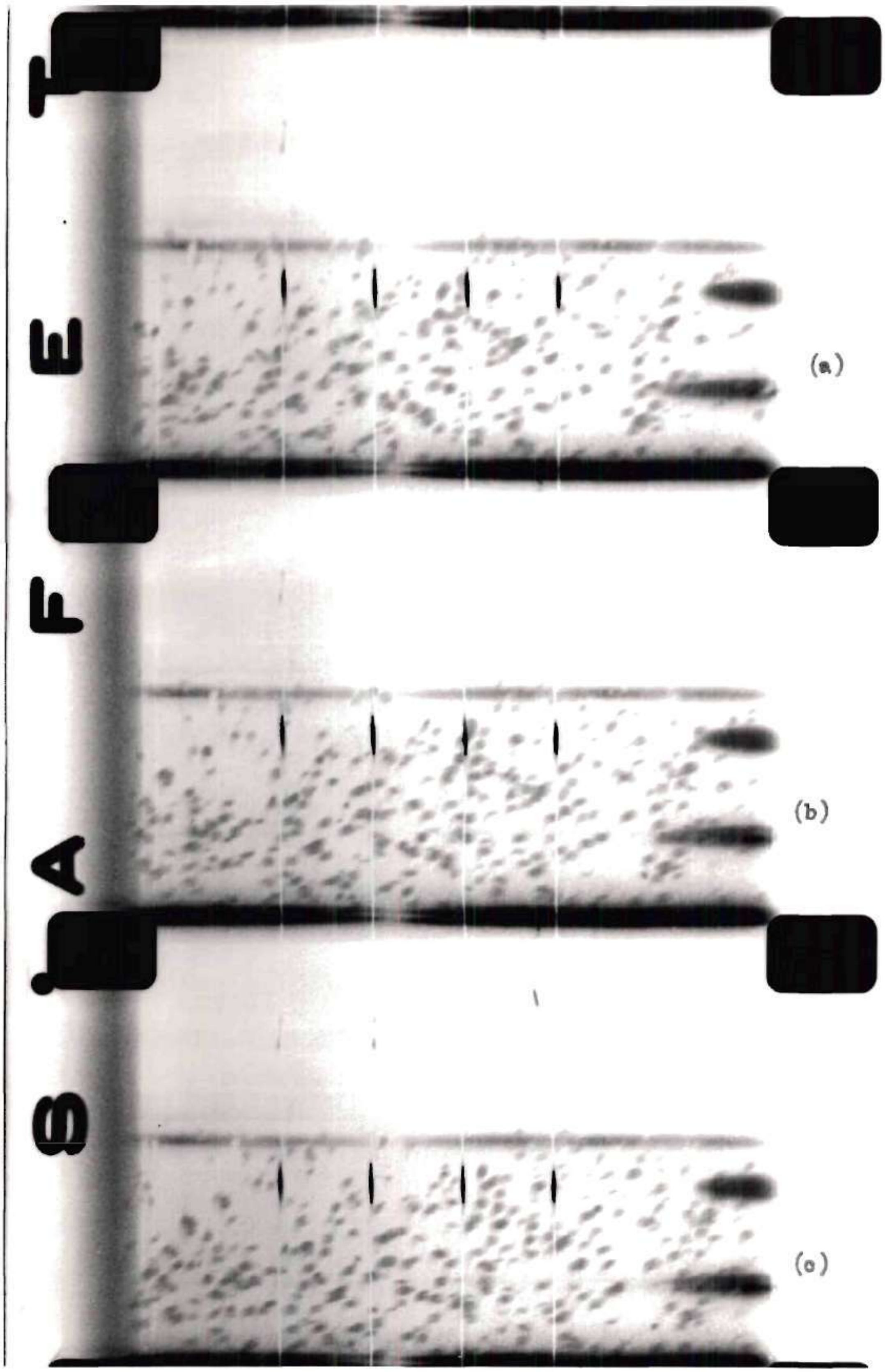

Figure 8. A Sequence of High Speed Photographs of Particles With an Average Velocity of Sixty-two Feet Per Second.

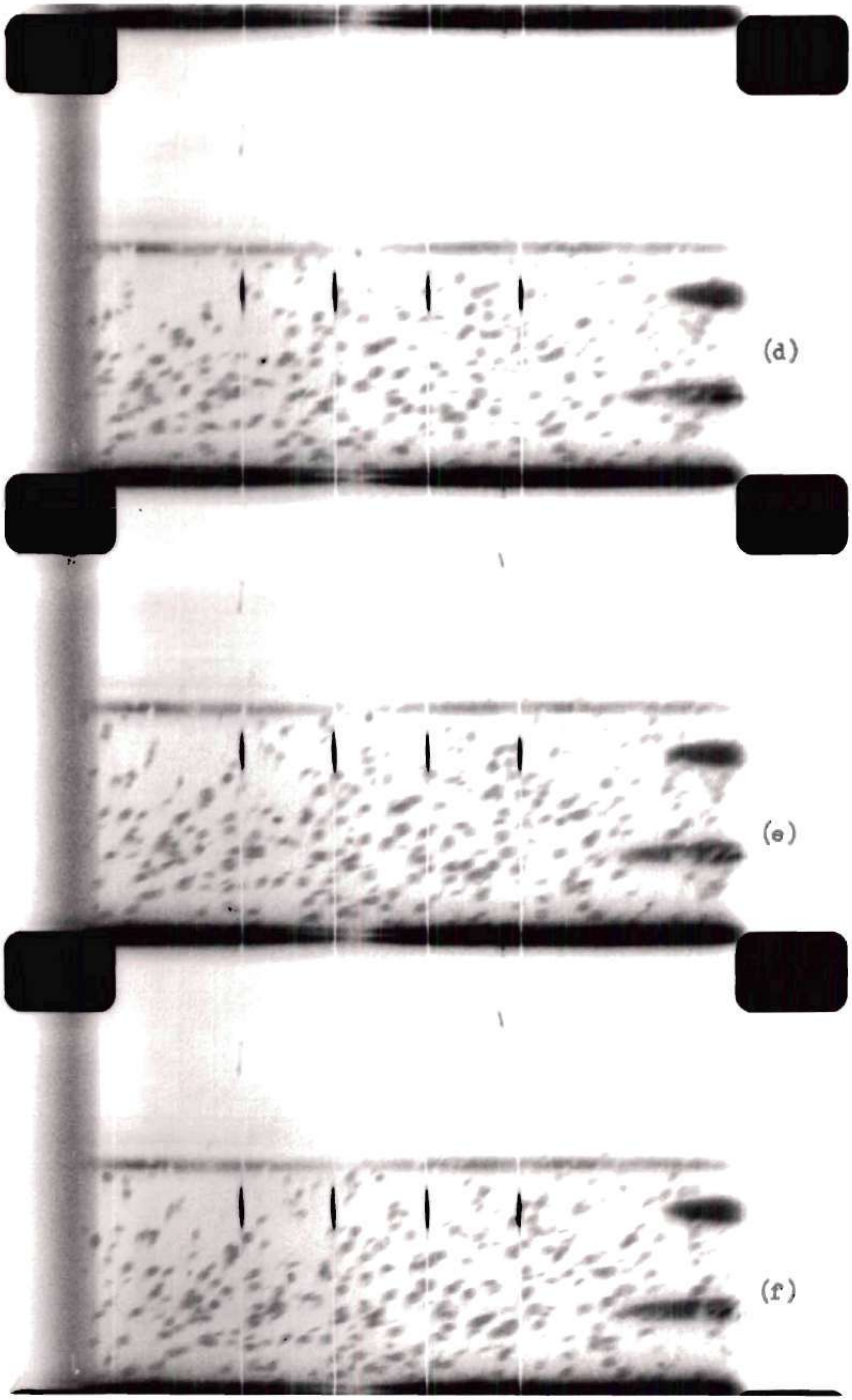

Figure 9. A Sequence of High Speed Photographs of Particles With an Average Velocity of Sixty-two Feet Per Second.

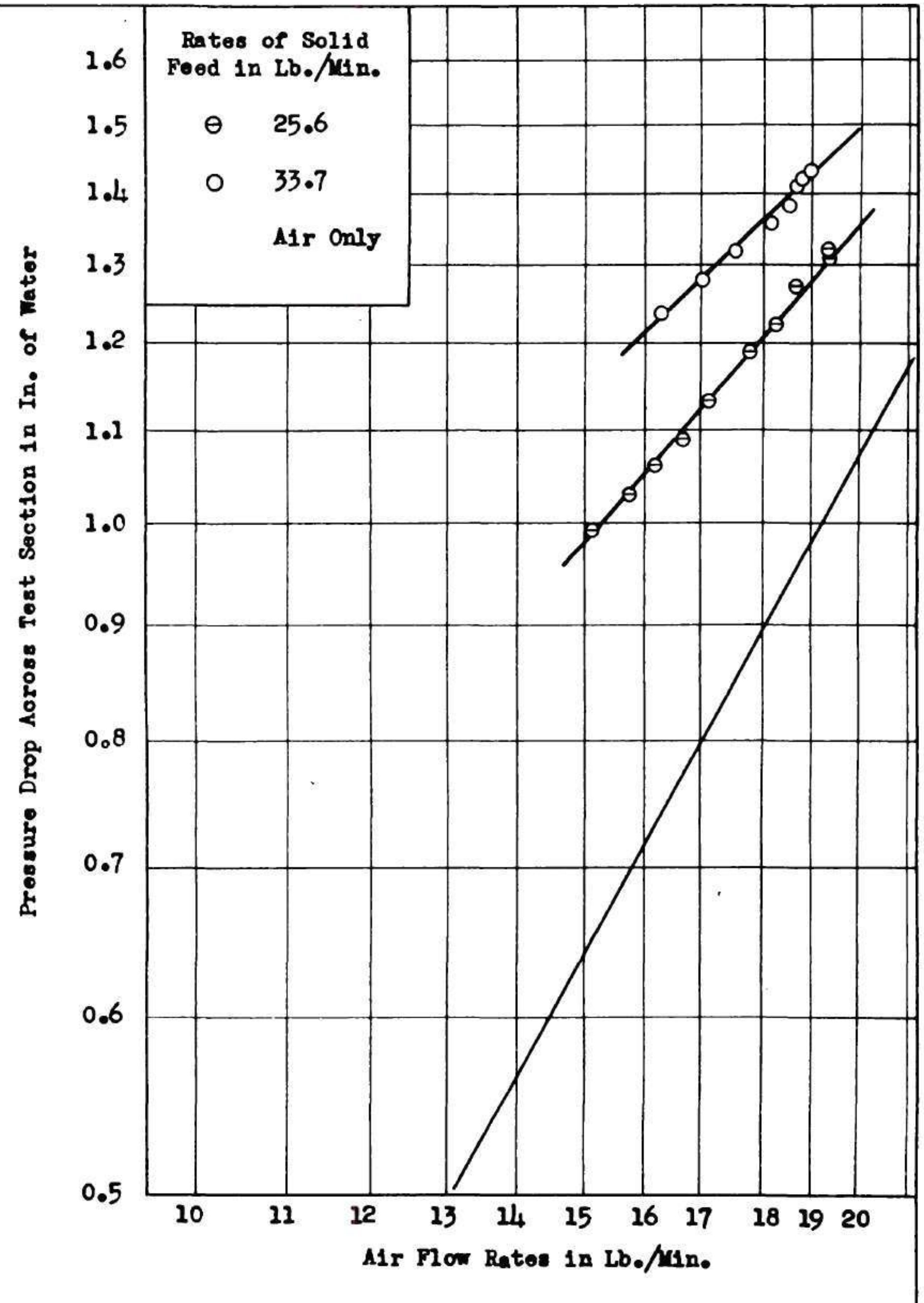

Figure 10. Pressure Drop - Air Flow Relationships for Cottonseed in Roughened Pipe.

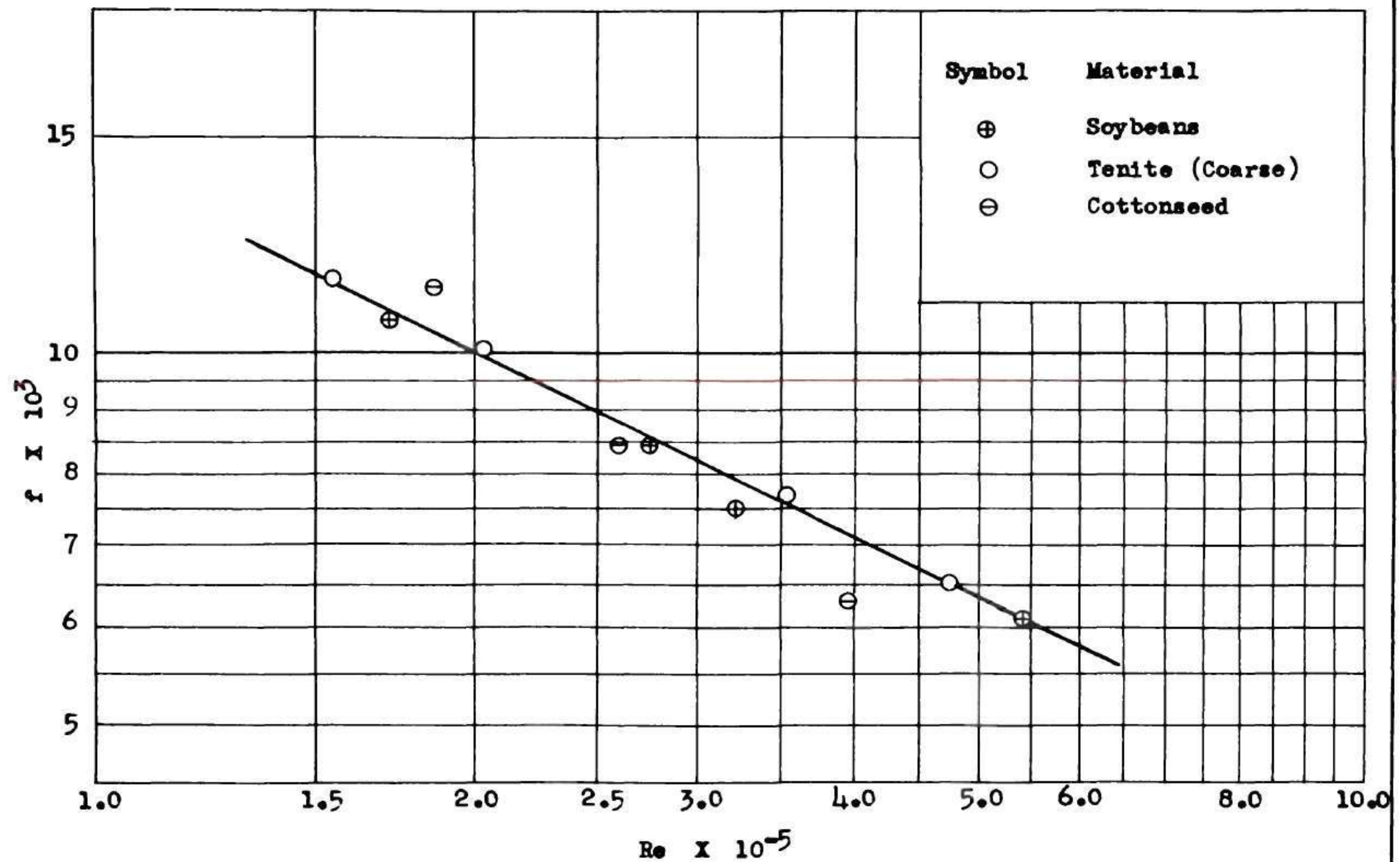

Figure 11. Modified Friction Factor and Reynolds Number for Critical Velocities of Soybeans, Tenite, and Cottonseed.

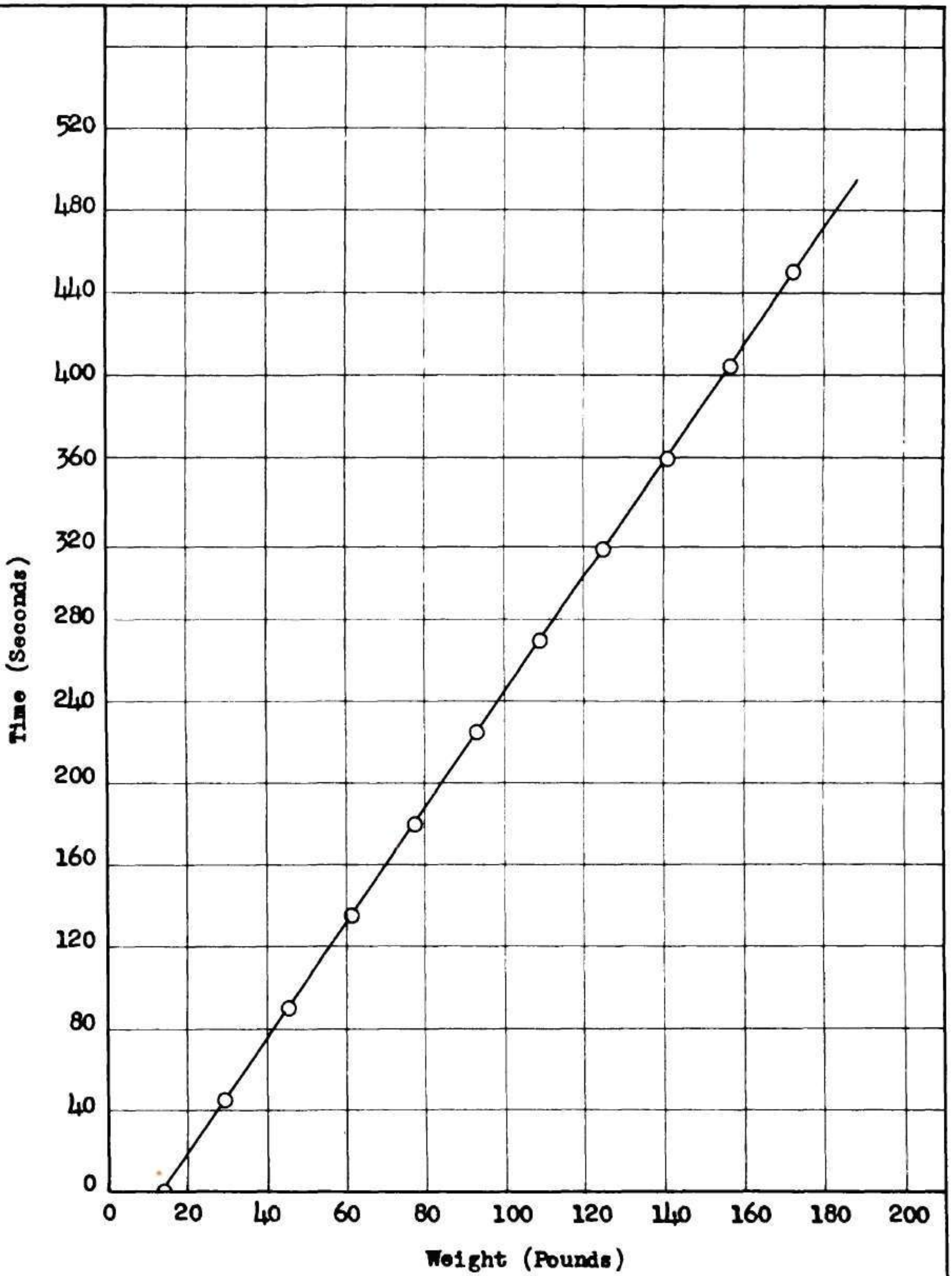

Figure 12. Typical Calibration Curve Obtained During Long Feeder Runs.

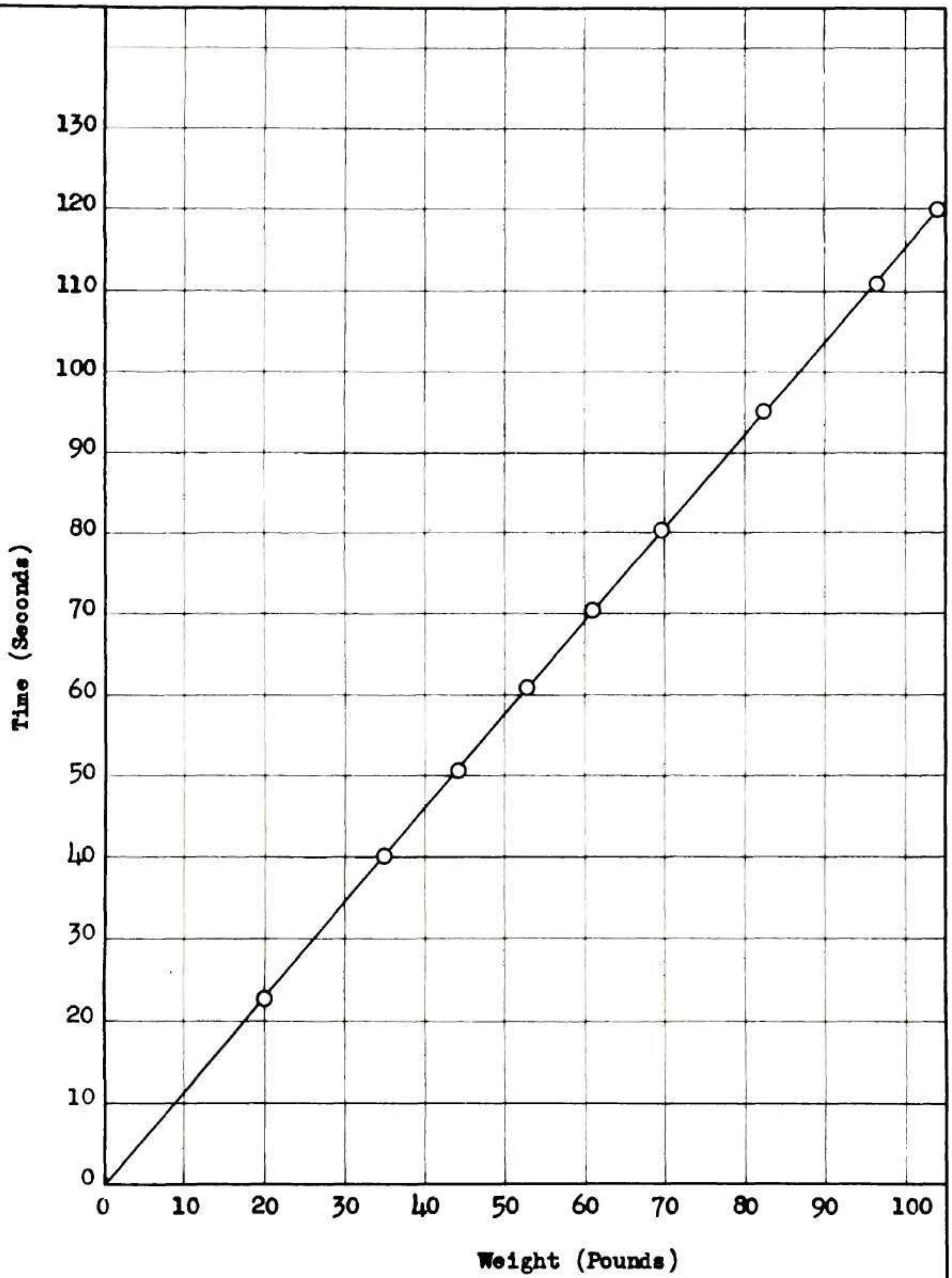

Figure 13. Typical Calibration Curve Obtained During Short Feeder Runs.